Proceedings of the 2025 SICE Festival with Annual Conference
September 9-12, 2025, Chiang Mai, Thailand

Approach to Power System Reduction Preserving small-signal Stability based on Energy Functions

Taku Nishino[1†], Jigen Koizumi[1] and Takayuki Ishizaki[1]

[1]Department of Systems and Control Engineering, Institute of Science Tokyo, Tokyo, Japan
(Tel: +81-3-5734-2589; E-mail: ta.nishino@lim.sc.e.titech.ac.jp, koizumi.j@lim.sc.e.titech.ac.jp, ishizaki@sc.e.titech.ac.jp)

Abstract: In this study, we propose a theoretical framework for assessing the small-signal stability of power systems by extending the concept of passivity from an energy function perspective. Specifically, we formulate new necessary and sufficient conditions for the small-signal stability of power systems in an analytically tractable form. Furthermore, based on this insight, we establish a systematic methodology for reducing a sub-area within the power system to an equivalent single synchronous generator model while rigorously preserving its small-signa stability characteristics. This proposal can contribute to the reduction of computational burden, elimination of conservatism, and improvement of analytical accuracy in the stability analysis of large-scale, complex power systems, thereby supporting more reliable decision-making in the planning and operation of future power systems.

Keywords: Power System, Small-signal stability, Kron reduction, Passivity

1. INTRODUCTION

Research on analyzing the small-signal stability of power systems has been actively pursued for many years[1]. Classical eigenvalue analysis, based on the state-space representation of the entire system, offers necessary and sufficient conditions for stability[2]. However, the necessity of centralized data from a substantial number of geographically dispersed components renders it progressively impractical for contemporary power systems. This challenge has prompted interest in analytical approaches capable of addressing large-scale power systems composed of numerous nodes by enabling their decomposition into manageable subsystems. Techniques such as passivity analysis, focusing on energy functions, and frequency-domain methods have significantly contributed to this advancement, allowing overall system stability to be inferred from the properties of these subsystems. While these decompositional approaches are crucial for analyzing complex, multi-node interactions, they generally provide only sufficient conditions for stability, introducing a degree of conservatism [3-5]. This conservatism, inherent in sufficient conditions, has the potential to compromise operational flexibility and economic efficiency.

In this study, we consider a power system consisting of a mix of IBRs, synchronous generators, and constant power loads, assuming no transmission losses for simplicity. The number of nodes and the topology of the considered power system are arbitrary.

The main contributions of this research are the proposal of a method to convert a system area to an equivalent synchronous generator model from the perspective of small-signal stability. To achieve these contributions, this research utilizes the concept of passivity as a core analytical tool. Power systems are generally described as differential-algebraic equation systems. In conventional analysis, stability is often discussed after eliminating algebraic variables and converting the system into an ordinary differential equation system. Our research group's prior work demonstrated that the convexity of the storage function in passivity analysis is equivalent to the small-signal stability of the power system represented by an ODE system [6]. Furthermore, we derived conditions under which the convexity of a Hessian matrix, constructed solely from algebraic variables by reducing the state variables of the DAE system to the algebraic side, has a necessary and sufficient relationship with the system's small-signal stability [7].

This research extends this prior work. While the preceding study proposed a method for evaluating stability at the level of algebraic variables by reducing equipment nodes when considering a power system composed of bus nodes and equipment nodes, this study further develops this approach. We also target bus node groups within a specific area of the system for reduction, formulating it in such a way that the necessary and sufficient conditions for small-signal stability are maintained even in the reduced system. Then, we establish a method to determine the parameters of an equivalent synchronous generator model by working backward to satisfy the characteristics of the resulting Hessian matrix of the reduced system. This is expected to enable the representation of areas containing complex groups of inverters and loads with a classical generator model that is analytically tractable, thereby leading to more efficient stability analysis of the entire system and facilitating intuitive understanding.

2. POWER SYSTEM MODEL

2.1. Transmission Network Model

Consider a transmission network with N buses. We assume that each bus is connected to a generator, load, or other device, without loss of generality. Let $b_{ij} > 0$ be the susceptance of the transmission line connecting Buses

† Taku Nishino is the presenter of this paper.

i and j. The susceptance matrix $B \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ is given by

$$B_{ij} = \begin{cases} b_{ij}, & i \neq j \\ -\sum_{j=1}^{N} b_{ij}, & i = j. \end{cases} \tag{1}$$

Let $\boldsymbol{I}_i \in \mathbb{C}$ and $\boldsymbol{V}_i \in \mathbb{C}$ be the current and voltage phasors at Bus i, respectively. Assuming zero line conductances, i.e., lossless, the power balance at each bus is given as

$$\begin{cases} P_i = \sum_{j=1}^{N} B_{ij} V_i V_j \sin(\theta_i - \theta_j) \\ Q_i = \sum_{j=1}^{N} -B_{ij} V_i V_j \cos(\theta_i - \theta_j) \end{cases} \quad i \in \mathbb{N} \tag{2}$$

where $\mathbb{N}$ is the label set of buses, V_i and θ_i are the magnitude and phase of $\boldsymbol{V}_i$. Furthermore, P_i and Q_i are the active and reactive powers defined as the real and imaginary parts of $\boldsymbol{V}_i \overline{\boldsymbol{I}_i}$, respectively. The power balance equation in (2) can be simply written as

$$0 = g(\theta, V, P, Q). \tag{3}$$

2.2. Synchronous Generator Model

In this paper, we consider a mathematical model of a synchronous generator called the one-axis model [1]. The equation of state of the one-axis model is expressed as

$$\begin{cases} \dot{\delta}_i = \omega_0 \omega_i \\ M_i \dot{\omega}_i = -D_i \omega_i - P_i + P_{\mathrm{mi}}^{\star} \\ \tau_{\mathrm{di}} \dot{E}_{\mathrm{qi}} = -E_{\mathrm{qi}} - (X_{\mathrm{di}} - X'_{\mathrm{di}}) I_{\mathrm{di}} + V_{\mathrm{fdi}}^{\star} \end{cases} \tag{4a}$$

where $E_{\mathrm{qi}} \in \mathbb{R}$ is the internal voltage of the generator with respect to the field winding on the q axis, $\delta_i \in \mathbb{R}$ is the the rotor deviation relative to the coordinate system rotating at the reference angular frequency $\omega_0 \in \mathbb{R}$, and $\omega_i \in \mathbb{R}$ is the angular frequency deviation from ω_0. In addition, M_i is the inertia constant, D_i is the damping coefficient, τ_{di} are the internal voltage time constants, $X_{\mathrm{di}}, X_{\mathrm{qi}}$ are synchronous reactances, X'_{di} is the transient reactance, $P_{\mathrm{mi}}^{\star}$ is the mechanical input, and $V_{\mathrm{fdi}}^{\star}$ is the field voltage.

The equations for the connection between the generator and the bus are given as equations for the currents I_{di} and I_{qi} flowing into the bus along the d-axis and q-axis, respectively

$$I_{\mathrm{di}} = \tfrac{1}{X'_{\mathrm{di}}}(E_{\mathrm{qi}} - V_{\mathrm{qi}}), \quad I_{\mathrm{qi}} = \tfrac{1}{X'_{\mathrm{qi}}} V_{\mathrm{di}} \tag{4b}$$

where $V_{\mathrm{qi}}, V_{\mathrm{di}}$ are defined as

$$V_{\mathrm{qi}} = V_i \cos(\delta_i - \theta_i), \quad V_{\mathrm{di}} = V_i \sin(\delta_i - \theta_i) \tag{4c}$$

Expressing equation (4b) in terms of the active power P_i ans reactive power Q_i that the generator supplies to the bus, we obtain

$$\begin{cases} P_i = \frac{E_{\mathrm{qi}}}{X'_{\mathrm{di}}} V_{\mathrm{di}} - \frac{E_{\mathrm{di}}}{X'_{\mathrm{qi}}} V_{\mathrm{qi}} + \left(\frac{1}{X'_{\mathrm{qi}}} - \frac{1}{X'_{\mathrm{di}}} \right) V_{\mathrm{di}} V_{\mathrm{qi}} \\ Q_i = \frac{E_{\mathrm{qi}}}{X'_{\mathrm{di}}} V_{\mathrm{qi}} + \frac{E_{\mathrm{di}}}{X'_{\mathrm{qi}}} V_{\mathrm{di}} - \left(\frac{V_{\mathrm{di}}^2}{X'_{\mathrm{qi}}} + \frac{V_{\mathrm{qi}}^2}{X'_{\mathrm{di}}} \right). \end{cases} \tag{4d}$$

The dynamic characteristics of equation (4) can be formally expressed as:

$$\begin{cases} \dot{x}_i = f_i(x_i, v_i; u_i^{\star}) \\ w_i = h_i(x_i, v_i; u_i^{\star}), \end{cases} \tag{5}$$

where x_i is a variable summarizing the internal states, and u_i is a constant summarizing the mechanical input and field voltage, and

$$v_i := (\theta_i, V_i), \quad w_i := (P_i, Q_i).$$

2.3. VSG Inverter Model

Here, we will discuss the Virtual Synchronous Generator (VSG) model as an example of an inverter power supply model. The VSG Inverter model is designed to mimic the dynamic characteristics of a simplified synchronous generator model, often referred to as the classical model. The classical model of a synchronous generator is representeed as

$$\begin{cases} \dot{\delta}_i = \omega_0 \omega_i \\ M_i \Delta\dot{\omega}_i = -D_i \omega_i - P_i + P_{\mathrm{mi}}^{\star} \end{cases} \tag{6a}$$

The connection equation with bus(4b) is simplified to:

$$I_{\mathrm{di}} = \tfrac{1}{X_{\mathrm{di}}}(V_{\mathrm{fdi}}^{\star} - V_{\mathrm{qi}}), \; I_{\mathrm{qi}} = \tfrac{1}{X_{\mathrm{qi}}} V_{\mathrm{di}} \tag{6b}$$

In the form of equation (4d), this becomes:

$$\begin{cases} P_i = \frac{V_{\mathrm{fdi}}^{\star}}{X_{\mathrm{di}}} V_{\mathrm{di}} + \left(\frac{1}{X_{\mathrm{qi}}} - \frac{1}{X_{\mathrm{di}}} \right) V_{\mathrm{di}} V_{\mathrm{qi}} \\ Q_i = \frac{V_{\mathrm{fdi}}^{\star}}{X_{\mathrm{di}}} V_{\mathrm{qi}} - \left(\frac{V_{\mathrm{di}}^2}{X_{\mathrm{qi}}} + \frac{V_{\mathrm{qi}}^2}{X_{\mathrm{di}}} \right) \end{cases} \tag{6c}$$

The dynamics of equation (6) can also be expressed in the form of equation (5).

2.4. Whole Power System Model

Using the forms of (3) and (5), the entire power system model is obtained as a nonlinear differential-algebraic equation system

$$\begin{cases} \dot{x} = f(x, v; u^{\star}) \\ 0 = g(v, h(x, v; u^{\star})) \end{cases} \tag{7}$$

3. EXISTING RESEARCH

In existing research, we consider the Hessian H of the energy function $U_g(x, v)$ of the equipment group subsystem. This Hessian H is used in passivity analysis with respect to the state variables x and algebraic variables v, and is defined as

$$H = \begin{bmatrix} H_{xx} & H_{xv} \\ H_{vx} & H_{vv} \end{bmatrix}. \tag{8}$$

Next, let L be the Hessian of the transmission line subsystem energy function $U_{br}(v)$ with respect to v. The linearized model of the system, assumed to be defined by (7), is then described as

$$\begin{aligned}\dot{x} &= R(H_{xx}x + H_{xv}v + u)\\ 0 &= H_{vx}x + (H_{vv} + L)v\end{aligned} \tag{9}$$

Eliminating the algebraic variable v, the state equation is expressed as:

$$\dot{x} = -R\underbrace{\{H_{xx} - H_{xv}(H_{vv}+L)^{-1}H_{vx}\}}_{H_{ode}}x + Ru$$

Existing research has shown that R satisfies $R + R^{\mathrm{T}} \succeq 0$. By applying this property and inertia theorem, the small-signal stability of the system is equivalent to the matrix H_{ode} being positive definite. By proving that $H_{vv}+L$ always holds and applying the Schur complement lemma, it was demonstrated that the positive definiteness of the entire Hessian in equation (8) is equivalent to the small-signal stability of the system. Furthermore, by employing the Schur complement lemma, it was deduced that $L + (H_{vv} - H_{vx}H_{xx}^{-1}H_{xv}) \succeq 0$ and $H_{xx} \succ 0$ are the necessary and sufficient conditions for the small-signal stability of the power system. Formulating this condition results in

$$Q_i + \frac{V_{\mathrm{q}i}^2}{X_{\mathrm{q}i}} + \frac{V_{\mathrm{q}i}^2}{X_{\mathrm{d}i}} \succ 0, \tag{10a}$$

$$\mathsf{diag}\big(\mathit{\Gamma}_i(v_i^\star, w_i^\star; X_i)\big) + L \succeq 0 \tag{10b}$$

where the submatrix $\mathit{\Gamma}_i$ are defined as in (11).

$$\mathit{\Gamma}_i(v_i, w_i; X_i) := \begin{bmatrix} 0 & 0 \\ 0 & \rho_i(v_i, w_i, X_i)\end{bmatrix} \tag{11}$$

The details of ρ are omitted, but it can be expressed as a function of v, w [7]. L can be expressed as

$$L_{ij} = \begin{cases} \sum_{j\neq i}^{N}\begin{bmatrix} B_{ij}V_iV_j\cos\theta_{ij} & B_{ij}V_j\sin\theta_{ij} \\ B_{ij}V_j\sin\theta_{ij} & B_{ij}\end{bmatrix}, i = j \\ \begin{bmatrix} -B_{ij}V_iV_j\cos\theta_{ij} & B_{ij}V_i\sin\theta_{ij} \\ -B_{ij}V_j\sin\theta_{ij} & -B_{ij}\cos\theta_{ij}\end{bmatrix}, i \neq j \end{cases} \tag{12}$$

4. PROPOSED METHOD

The method presented in section 3 shows that the small-signal stability of a power system can be characterized by the positive definiteness of the Hessian of only the algebraic variables. The advantage of this method is that the network structure of the system can be preserved.

In this section, we will develop the theory from the previous section and introduce a method to assess small-signal stability by reducing an area within the system to a single equivalent synchronous generator. In the following, we will compare the conditions under which the Hessian of equation (10b) becomes positive definite for the two power systems shown in Fig 1. It should be noted that the discussion will proceed under the assumption that the conditions that each component must satisfy, as given by equation (10a), are met.

4.1. Two areas model

First, let us consider the model on the left side of Fig 1. Nodes 1 to i are denoted as Area Ω_1, and nodes $i+1$ to N are denoted as Area Ω_2. Furthermore, the j-th node in Area Ω_2 and the i-th node in Area Ω_1 are connected by a transmission line. If we set the left side of equation (10b) as $H_{\mathrm{alg}} = \Gamma + L$, then H_{alg} is expressed as

$$H_{\mathrm{alg}} = \left[\begin{array}{ccc|ccc} H_{\mathrm{alg}11} & \cdots & H_{\mathrm{alg}1i} & 0 & 0 & 0 \\ \vdots & & \vdots & 0 & 0 & 0 \\ H_{\mathrm{alg}i1} & \cdots & H_{\mathrm{alg}ii} & H_{\mathrm{alg}ij} & 0 & 0 \\ \hline 0 & 0 & H_{\mathrm{alg}ji} & H_{\mathrm{alg}jj} & \cdots & H_{\mathrm{alg}jN} \\ 0 & 0 & 0 & \vdots & & \vdots \\ 0 & 0 & 0 & H_{\mathrm{alg}Nj} & \cdots & H_{\mathrm{alg}NN}\end{array}\right]. \tag{13}$$

Using Schur's complement, the condition for $H_{\mathrm{alg}} \succeq 0$ is

$$H_{\mathrm{alg}\tilde{j}\tilde{j}} \succ 0 \tag{14a}$$

$$\left[\begin{array}{ccc|c} H_{\mathrm{alg}11} & \cdots & H_{\mathrm{alg}1i} & 0 \\ \vdots & & \vdots & 0 \\ H_{\mathrm{alg}i1} & \cdots & H_{\mathrm{alg}ii} & H_{\mathrm{alg}ij} \\ \hline 0 & 0 & H_{\mathrm{alg}ji} & H_{\mathrm{alg}jj}/H_{\mathrm{alg}\tilde{j}\tilde{j}}\end{array}\right] \succeq 0 \tag{14b}$$

where, $\tilde{j}$ is the index set from $i+1$ to N without j, $H_{\mathrm{alg}ij}$ is the same as L_{ij} shown in equation (12), and $H_{\mathrm{alg}jj}/H_{\mathrm{alg}\tilde{j}\tilde{j}}$ can be represented as

$$\begin{aligned} &H_{\mathrm{alg}jj}/H_{\mathrm{alg}\tilde{j}\tilde{j}} \\ &= \begin{bmatrix} B_{ij}V_iV_j\cos\theta_{ij} & B_{ij}V_j\sin\theta_{ij} \\ B_{ij}V_j\sin\theta_{ij} & B_{ij} + \rho(v_{\Omega_2}, w_{\Omega_2})\end{bmatrix}. \end{aligned} \tag{15}$$

The ρ can be expressed as a function that depends on the steady-state power flow of the area $\Omega 2$, but is omitted here for page convenience.

4.2. One area and a synchronous generator model

On the other hand, if we consider the Hesse matrix of the energy function when a one-axis synchronous generator model is added to a system consisting only of Area 1 of the same power system, we can derive condition as

$$\left[\begin{array}{ccc|c} H_{\mathrm{alg}11} & \cdots & H_{\mathrm{alg}1i} & 0 \\ \vdots & & \vdots & 0 \\ H_{\mathrm{alg}i1} & \cdots & H_{\mathrm{alg}ii} & H_{x_iv_i} \\ \hline 0 & 0 & H_{x_iv_i}^{\mathrm{T}} & H_{x_ix_i}\end{array}\right] \succeq 0, \tag{16}$$

where

$$\begin{aligned} H_{x_iv_i} &= \begin{bmatrix} -\frac{E_{\mathrm{q}i}V_i}{X_i'}\cos(\delta_i - \theta_i) & \frac{V_i}{X_i'}\sin(\delta_i - \theta_i) \\ -\frac{E_{\mathrm{q}i}}{X_i'}\sin(\delta_i - \theta_i) & -\frac{1}{X_i'}\cos(\delta_i - \theta_i)\end{bmatrix} \\ H_{x_ix_i} &= \begin{bmatrix} \frac{V_iE_{\mathrm{q}i}}{X'}\cos(\delta_i - \theta_i) & -\frac{V_i}{X_i'}\sin(\delta_i - \theta_i) \\ \frac{V_i}{X_i'}\sin(\delta_i - \theta_i) & \frac{1}{X_i'} + \frac{1}{X_i - X_i'}\end{bmatrix}. \end{aligned} \tag{17}$$

Comparing equations (12)(15) and (17), the Hessian of the two cases become equivalent if the parameters of

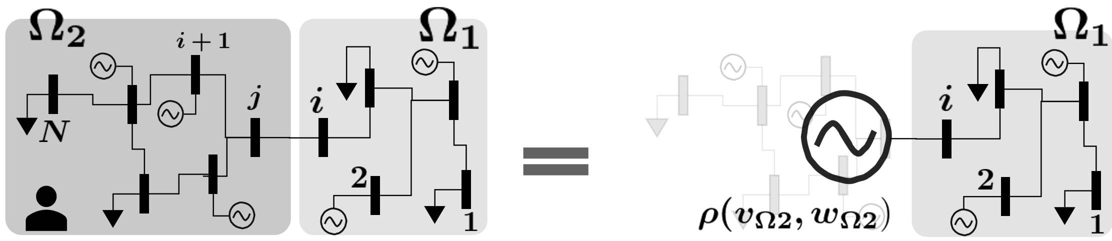

Fig. 1 The concept of this research is to propose a reduction method in which the above two power systems are equivalent in terms of small-signal stability.

the connected one-axis synchronous generator have the following relationship:

$$\frac{1}{X_i'} = B_{ij}, \quad \frac{1}{X_i - X_i'} = \rho(v_{\Omega 2}, w_{\Omega 2})$$

From the above, if we consider a connected synchronous generator model described by

$$\begin{cases} \dot{\delta}_i = \omega_0 \omega_i \\ M_i \dot{\omega}_i = -D_i \omega_i - P_i + P_{\mathrm{mi}}^{\star} \\ \tau_{\mathrm{di}} \dot{E}_{\mathrm{qi}} = -E_{\mathrm{qi}} - \dfrac{1}{\rho(v_{\Omega 2}, w_{\Omega 2})} I_{\mathrm{di}} + V_{\mathrm{fdi}}^{\star} \end{cases} . \tag{18}$$

$$I_{\mathrm{di}} = B_{ij}(E_{\mathrm{qi}} - V_{\mathrm{qi}}), \quad I_{\mathrm{qi}} = B_{ij} V_{\mathrm{di}}, \tag{19}$$

then it can be said that the positive definiteness of the Hessian of the energy function of the model considered in Section 4.2 and the Hessian of equation (14b) are consistent.

5. CONCLUSION

In summary, existing research formulated the small-signal stability of a power system as equation (10). Equation (10a) can be discussed for each component individually, but equation (10b) required a collective analysis of the system-wide component characteristics and steady-state power flow conditions. This research has shown that the condition of equation (10b) proves to be equivalent to the small-signal stability of the reduced model detailed in equations (14a) and (16). The condition of equation (14a) can be analyzed using only information within Area Ω2. Then, by replacing it with a synchronous generator model according to equation (18) based on the ρ_i obtained from these analyses, it becomes possible to discuss the small-signal stability of the entire system.

One of the contributions of this research is the aspect of security. In recent years, while the stability of power systems has become increasingly important, disclosing all internal system data also carries the risk of security vulnerabilities. By utilizing the method of this research, aggregators in each area of the system only need to maintain the conditions that each component or area must satisfy according to equations (10a) and (14a), and only need to disclose the scalar quantity ρ to other areas.

As future work, we consider a more detailed model. Specifically, while this study considered a model that ignores transmission losses, we aim to extend it to a form that considers transmission losses. Furthermore, we would also like to extend the inverter model to include detailed dynamics such as filters.

6. ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number JP24K07534, and is supported by JST SPRING, Grant Number JPMJSP JPMJSP2180.

REFERENCES

[1] P. Kundur, Power system stability and control. Tata McGraw-Hill Education, 1994.

[2] Y. Zhang, H. Liu, P. Song, and L. Wu, "Small-signal stability analysis of multiple grid-connected virtual synchronous generators," in 2018 China International Conference on Electricity Distribution (CICED), 2018, pp. 2034–2038.

[3] T. Stegink, C. De Persis, and A. van der Schaft, "A unifying energy-based approach to stability of power grids with market dynamics," Automatic Control, IEEE Transactions on, vol. 62, no. 6, pp. 2612–2622, 2017.

[4] Z. Wang, F. Liu, J. Z. Pang, S. H. Low, and S. Mei, "Distributed optimal frequency control considering a nonlinear network-preserving model," IEEE Transactions on Power Systems, vol. 34, no. 1, pp. 76–86, 2018.

[5] L. Huang, D. Wang, X. Wang, H. Xin, P. Ju, K. H. Johansson, and F. Dörfler, "Gain and phase: Decentralized stability conditions for power electronics-dominated power systems," IEEE Transactions on Power Systems, vol. 39, no. 6, pp. 7240–7256, 2024.

[6] T. Ishizaki, T. Nishino, and A. Chakrabortty, "Equilibrium-independent passivity of power systems: A link between classical and two-axis synchronous generator models," 2024. [Online]. Available: https://arxiv.org/abs/2304.00987

[7] T. Nishino, Y. Onishi, and T. Ishizaki, "Small-signal stability condition of inverter-integrated power systems: Closed-form expression by stationary power flow variables," arXiv preprint arXiv:2503.20276, 2025.