

Ultrasound Surface Waveguide with Periodic Comb-like Structure

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Abstract: In this paper, we show that a plane plate with a periodic comb-like structure on its surface functions as a surface waveguide for ultrasonic waves. In the waveguide, ultrasonic waves propagate in the tangential direction of the plane but are localized in the normal direction. The waveguide function is confirmed by numerical simulations.

Keywords: Ultrasound, acoustic waveguide, metamaterial.

1. INTRODUCTION

In this paper, we show that a plane plate with a periodic comb-like structure on its surface functions as a surface waveguide for ultrasonic band waves. For the waveguide, the ultrasound propagates in the tangential direction of the plane but is localized and attenuate exponentially in the normal direction. In radio engineering, it is known that terahertz surface plasmons can propagate along metal surfaces with comb-like structures [1]. Below, we use a similar argument to show that ultrasonic waves can propagate through air, and confirm this with numerical simulations.

2. PROPAGATION MODES ON COMB-STRUCTURED SURFACE

The dimensions and coordinate system of a perfect reflector with a 1-D comb-shaped surface structure are shown in Fig. 1. The region $z < 0$ is assumed to be in air, the region $0 < z < h$ is a comb-shaped structure, and the region $z > h$ is a perfect reflector. In the y direction (perpendicular to the plane of the paper), the reflector structure and sound field are uniform. The span d of the periodic structure is assumed to be sufficiently short compared to the wavelength λ of the sound wave.

Now, suppose that a sinusoidal plane wave with wave vector $\mathbf{k} = (k_x, 0, k_z)$ is incident on a reflector and the reflected wave is composed of only the 0-th order diffracted wave. In this case, if the sound field is expressed as a velocity potential, the incident wave φ^i and the reflected wave φ^r are respectively as follows:

$$\varphi^i = A_i e^{jk_x x} e^{jk_z z} e^{-j\omega t}, \quad (1)$$

$$\varphi^r = R A_i e^{jk_x x} e^{-jk_z z} e^{-j\omega t}, \quad (2)$$

where A_i and R are the amplitude of the incident wave and reflection coefficient, respectively.

Therefore, the sound field $\varphi^{(1)}$ in the region $z < 0$ is a superposition as follows:

$$\begin{aligned} \varphi^{(1)} &= \varphi^i + \varphi^r \\ &= A_i e^{jk_x x} (e^{jk_z z} + R e^{-jk_z z}) e^{-j\omega t}. \end{aligned} \quad (3)$$

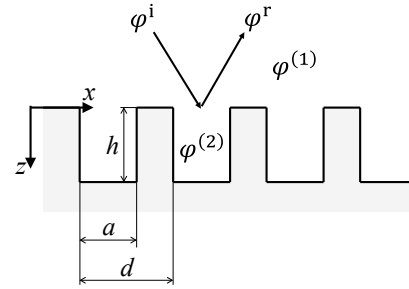


Fig. 1 Comb-like surface structure.

Here, the need for $\varphi^{(1)}$ to satisfy the wave equation leads to the following dispersion relationship:

$$k_x^2 + k_z^2 = k_0^2, \quad (4)$$

where $k_0 = \omega/c$ is a wave number in free air.

On the other hand, the sound field $\varphi^{(2)}$ in the cavity region of the comb-shaped structure is almost uniform in the x direction because $\lambda \gg d$, and can be expressed as follows:

$$\varphi^{(2)} = C^+ e^{jk_0 z} e^{-j\omega t} + C^- e^{-jk_0 z} e^{-j\omega t}, \quad (5)$$

where C^+ , C^- are constants determined by the boundary conditions.

The particle velocity v_z in the z direction can be expressed as follows from the definition of the velocity potential $v_z = \partial\varphi / \partial z$:

$$v_z = \begin{cases} jk_z A_i (e^{jk_z z} - R e^{-jk_z z}) e^{jk_x x} e^{-j\omega t} & (z < 0) \\ jk_0 (C^+ e^{jk_0 z} - C^- e^{-jk_0 z}) e^{-j\omega t} & (z > 0) \end{cases} \quad (6)$$

Furthermore, sound pressure p can be expressed as follows from the definition $p = -\rho \partial\varphi / \partial t$:

$$p = \begin{cases} j\omega \rho A_i (e^{jk_z z} + R e^{-jk_z z}) e^{jk_x x} e^{-j\omega t} & (z < 0) \\ j\omega \rho (C^+ e^{jk_0 z} + C^- e^{-jk_0 z}) e^{-j\omega t} & (z > 0) \end{cases} \quad (7)$$

where ρ is the density of air.

These coefficients A_i , A_r , C^+ , and C^- are constrained by the following boundary conditions (i)-(iii) between each domain.

(i) Continuum of particle velocity

At the boundary $z = 0$, v_z in Eq. (6) must be continuous, so that

$$k_z A_i (1 - R) e^{jk_x x} = k_0 (C^+ - C^-). \quad (8)$$

[†] Masahiro Fujiwara is the presenter of this paper.

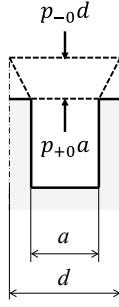


Fig. 2 Dynamic force balance at a boundary.

(ii) Fixed-end reflection at the bottom of a cavity

At the boundary $z = h$, v_z in Eq. (6) must be 0, so that

$$C^+ e^{jk_0 h} + C^- e^{-jk_0 h} = 0. \quad (9)$$

(iii) Dynamic equilibrium at macroscopic boundaries

Near the boundary $z = 0$, the unit cell of the periodic structure requires a dynamic force balance, which is expressed as $p_{-0}d = p_{+0}a$ on the top surface as shown in Fig. 2. The pressure p_{-0} , p_{+0} are the left and right limits at $z = 0$ of Eq. (7), respectively, so that

$$A_i(1 + R)e^{jk_x x d} = (C^+ + C^-)a. \quad (10)$$

From the above boundary conditions (8), (9), and (10), C^+ , C^- and A_i are eliminated, and the following reflection coefficient R is obtained:

$$R = -\frac{1 - j \frac{k_z a}{k_0 d} \tan(k_0 h)}{1 + j \frac{k_z a}{k_0 d} \tan(k_0 h)}. \quad (11)$$

Note that the absolute value of R is always 1 when k_z is a real number. The above determines the macroscopic behavior of this comb-shaped structure, but here we will determine the conditions under which the sound field is localized on the surface of the structure. That is, the conditions under which k_z is not a real number. In this case, the denominator of the amplitude reflectance becomes 0, so that

$$k_z = -\frac{1}{j \left(\frac{a}{d}\right) \tan(k_0 h)} k_0. \quad (12)$$

For the reflected wave φ^r , when k_z becomes a pure imaginary number like this, the attenuation distance $L = -1/jk_z$ in the z direction is as follows:

$$L = \frac{a}{k_0 d} \tan(k_0 h). \quad (13)$$

If $k_0 h \ll 1$, we can approximate $\tan(k_0 h) \approx k_0 h$ and the attenuation distance is

$$L \approx \frac{a}{d} h. \quad (14)$$

In other words, it is localized within a range approximately equal to the product of the height h of the cavity and the aperture ratio a/d . Therefore, the velocity potential of the reflected wave is expressed as follows:

$$\varphi^r = R A_i e^{z/L} e^{j(k_x x - \omega t)}. \quad (15)$$

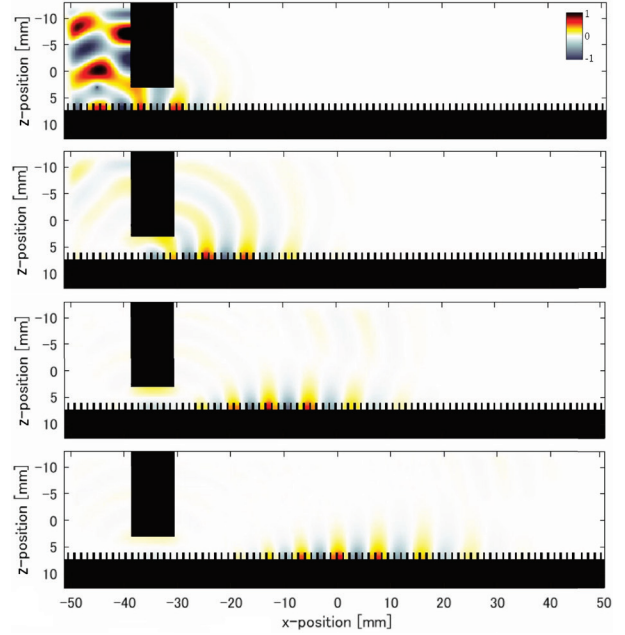


Fig. 3 Simulation results. The normalized sound pressure distribution is plotted every $50 \mu\text{s}$ from the top.

3. NUMERICAL SIMULATION

Numerical simulations were performed using MATLAB and the k-Wave toolbox [2]. The k-Wave toolbox numerically calculates the time evolution of a linear wave field in a lattice space based on the pseudo-spectral method.

For the simulation conditions, ultrasonic waves with a frequency of $f = 40 \text{ kHz}$ are incident on a comb-shaped structure with dimensions $d = 1.2 \text{ mm}$, $a = 0.8 \text{ mm}$, and $h = 1 \text{ mm}$. In order to block direct waves from a point sound source, the element plate is placed in the range of $-40 \text{ mm} < y < -30 \text{ mm}$ with a gap of 5 mm in height.

4. CONCLUSION AND FUTURE WORKS

In this paper, we propose a surface acoustic waveguide with a comb-like structure and present a basic numerical simulation result. In future work, we will carry out experimental verification and application.

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