

# Robust adaptive trajectory tracking of rigid body attitude based on ISS-TCLF

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**Abstract:** In this paper, we consider the trajectory tracking control of rigid body attitude subject to disturbance torque inputs. To achieve robust trajectory tracking, we employ the concept of input-to-state stability trajectory tracking control Lyapunov function (ISS-TCLF) and design a robust trajectory tracking controller based on the ISS-TCLF. We then introduce an adaptive disturbance estimator to mitigate the effect of disturbances more efficiently. The effectiveness of the proposed controllers is confirmed through numerical simulations.

**Keywords:** rigid body, attitude control, trajectory-tracking, input-to-state stability, control Lyapunov function (CLF), adaptive control

## 1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs), including multi-copters, are widely used in many applications such as aerial photography, surveying, and transportation. In such real-world applications, the robustness of the control systems to various environmental disturbances is critical. Specifically, precise attitude tracking control under such disturbances is essential to avoid crashes.

For this robust attitude tracking problem, various non-linear control methods such as sliding mode control [1, 2] or neural network-based approach [3] have been applied. In particular, many disturbance observer (DOB)-based controllers [4-7] have been proposed. The basic idea of the DOB is to estimate and cancel out the unknown time-varying disturbances directly.

In contrast to the DOB, adaptive control estimates unknown constant parameters. In recent years, adaptive control-based disturbance attenuation controllers [8, 9] have been proposed. The key idea here is to separate the unknown time-varying disturbances (parameters) into two parts: (i) the unknown constant term and (ii) the unknown residual time-varying term. The adaptive control compensates for the former, and the latter is treated by disturbance attenuation control. This approach is expected to be more robust to system modeling errors than disturbance observers.

Along this line of research, an adaptive attitude controller based on the control Lyapunov function (CLF) has been proposed for the attitude stabilization problem of rigid bodies [10]. This controller combines CLF-based adaptive control with the disturbance attenuation controller, which is based on the concept of input-to-state stability (ISS). However, we cannot directly apply this controller to the *trajectory tracking* problem considered in this paper because the standard CLF is a design tool for asymptotic stabilization.

The objective of this paper is to propose a robust adaptive trajectory controller for the rigid body attitude dynamics in the presence of disturbance torque. To do so,

we introduce the concept of input-to-state stability tracking Lyapunov function (ISS-TCLF) [9], which is an extension of tracking control Lyapunov function (TCLF) [11]. We first design a robust trajectory tracking controller, which guarantees the ISS, based on the ISS-TCLF. Then we extend this to the robust adaptive tracking controller by introducing an appropriate parameter estimation term. The effectiveness of the proposed controller is confirmed through numerical simulations.

The rest of this paper is organized as follows. Section 2 introduces some definitions and fundamental results used in the paper. The problem considered in this paper is formulated in Section 3. In Section 4, we design a robust trajectory tracking controller based on the ISS-TCLF and validate its effectiveness through numerical simulations in Section 5. We then propose the robust adaptive trajectory tracking controller in Section 6. Finally, a brief conclusion is given in Section 7.

## 2. PRELIMINARIES

### 2.1. Unit quaternions [12, 13]

In this paper, we characterize the attitude of rigid bodies by using the unit quaternion  $q \in S^3 := \{q \in \mathbb{R}^4 \mid \|q\| = 1\}$ . As is well-known, any rotation matrix  $R \in SO(3) := \{R \in \mathbb{R}^{3 \times 3} \mid R^T R = I, \det R = 1\}$  is characterized by the unit quaternion by the following map  $\mathcal{R}$ :

$$R = \mathcal{R}(q) = I + 2r_0 S(r) + 2(S(r))^2, \quad (1)$$

where  $S(r)$  is a skew-symmetric matrix defined as follows:

$$S(r) = \begin{pmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{pmatrix}. \quad (2)$$

For two given unit quaternions  $q_a = (r_{0a} \ r_a^T)^T$  and  $q_b = (r_{0b} \ r_b^T)^T$ , the quaternion multiplication is defined as follows:

$$q_a \otimes q_b = \begin{bmatrix} -r_{0a}r_{0b} + r_a^T r_b \\ -r_{0a}r_b - r_{0b}r_a - S(r_a)r_b \end{bmatrix}. \quad (3)$$

<sup>†</sup> Yasuyuki Satoh is the presenter of this paper.

The conjugate of a unit quaternion  $q = (r_0 \ r^\top)^\top$  is defined by  $q^{-1} := (r_0 \ -r^\top)^\top$ .

## 2.2. Robust trajectory tracking based on ISS-TCLF [9]

In this subsection, we introduce the basic definitions and results of robust trajectory tracking control of nonlinear systems based on input-to-state tracking control Lyapunov function (ISS-TCLF). Let us consider the following nonlinear control system:

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + h(x)d \\ &= f(x) + \sum_{i=1}^m g_i(x)u_i + \sum_{i=1}^l h_i(x)d_i,\end{aligned}\quad (4)$$

where  $d \in \mathbb{R}^l$  is an exogenous disturbance and the mappings  $g_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$  ( $i = 1, \dots, m$ ) and  $h_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$  ( $i = 1, \dots, l$ ) are assumed to be locally Lipschitz continuous.

We first introduce the standard trajectory tracking control problem for system (4) with  $d = 0$ . The following assumption is essential for trajectory tracking:

**Assumption 1.** Consider the nonlinear system  $\dot{x} = f(x) + g(x)u$  (i.e., system (4) with  $d = 0$ ). We suppose that a  $C^2$  desired trajectory  $x_d : [0, \infty) \rightarrow \mathbb{R}^n$  satisfying the following equation is given:

$$\dot{x}_d = f(x_d(t)) + g(x_d(t))u_r(t), \quad \forall t \geq 0, \quad (5)$$

where  $u_r : [0, \infty) \rightarrow \mathbb{R}^m$ ;  $t \mapsto [u_{r1}(t), \dots, u_{rm}(t)]^\top$  is the corresponding  $C^1$  reference input.

**Remark 1.** Note that Assumption 1 requires the desired trajectory to be realizable; in other words, the system itself must be able to generate  $x_d(t)$ .

Under Assumption 1, the trajectory tracking problem is formulated as follows:

**Definition 1** (Trajectory tracking). Consider the nonlinear system  $\dot{x} = f(x) + g(x)u$  (i.e., system (4) with  $d = 0$ ). Let  $x_d(t)$  and  $u_r(t)$  be the desired trajectory and the corresponding reference input, respectively, satisfying Assumption 1. Then, the trajectory tracking control problem is to design a time-varying continuous state feedback controller  $u = k(t, x)$  such that  
(A1) for any  $x_0 \in \mathbb{R}^n$ , the solution  $x(t)$  is uniformly bounded on  $[0, \infty)$ ,  
(A2) the tracking error  $e(t) := x(t) - x_d(t)$  converges to zero as  $t \rightarrow \infty$ .

By using the tracking error variable  $e(t)$  and the new input  $\tilde{u} := u - u_r(t)$ , we can obtain the following error dynamics:

$$\dot{e} = \tilde{f}(t, e) + \tilde{g}(t, e)\tilde{u}, \quad (6)$$

where

$$\begin{aligned}\tilde{f}(t, e) &:= f(e + x_d(t)) - f(x_d(t)) \\ &\quad + [g(e + x_d(t)) - g(x_d(t))]u_r(t),\end{aligned}\quad (7)$$

$$\tilde{g}(t, e) := g(e + x_d(t)). \quad (8)$$

Note that  $e(t) = 0$  is equivalent to  $x(t) = x_d(t)$ , and hence, the trajectory tracking problem reduces to asymptotic stabilization of  $e = 0$  of the time-varying error system (6).

Next, we consider the *robust* tracking problem when  $d \neq 0$ . In this case, the error system is given by

$$\dot{e} = \tilde{f}(t, e) + \tilde{g}(t, e)\tilde{u} + \tilde{h}(t, e)d, \quad (9)$$

where

$$\tilde{h}(t, e) := h(e + x_d(t)). \quad (10)$$

Just as the standard trajectory tracking corresponds to asymptotic stabilization of the error system (6), the robust trajectory tracking considered here corresponds to input-to-state stabilization of the error system (9) in the following sense:

**Definition 2** (Input-to-state stability). A state feedback  $\tilde{u} = \tilde{k}(t, x)$  said to input-to-state stabilizes the origin of system (9) if there exist  $\beta \in \mathcal{KL}$  and  $\chi \in \mathcal{K}$  such that

$$\|e(t)\| \leq \beta(e(0), t) + \chi \left( \sup_{0 \leq \tau \leq t} \|d(\tau)\| \right), \quad \forall t \geq 0. \quad (11)$$

To achieve robust trajectory tracking, i.e., input-to-state stabilizes  $e = 0$  of (9), the following input-to-state stability tracking control Lyapunov function (ISS-TCLF) plays an important role:

**Definition 3** (ISS-TCLF). A  $C^1$  differentiable function  $V : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be an input-to-state stable tracking control Lyapunov function (ISS-TCLF) for the error system (9) if the following conditions hold:

(B1) there exist positive definite proper functions  $\underline{V}, \bar{V} : \mathbb{R}^n \rightarrow \mathbb{R}$  such that

$$\underline{V}(e) \leq V(t, e) \leq \bar{V}(e), \quad \forall t \geq 0, \quad \forall e \in \mathbb{R}^n, \quad (12)$$

(B2) there exist  $\rho \in \mathcal{K}_\infty$  and a positive-definite function  $\tilde{Q} : \mathbb{R}^n \rightarrow \mathbb{R}$  such that

$$\begin{aligned}\|e\| &\geq \rho(\|d\|) \\ \Rightarrow \inf_{\tilde{u} \in \mathbb{R}^m} \left( \frac{\partial V}{\partial t} + L_{\tilde{f}}V + L_{\tilde{g}}V\tilde{u} + L_{\tilde{h}}Vd \right) &< \tilde{Q}(e).\end{aligned}\quad (13)$$

Based on the ISS-TCLF, we can input-to-state stabilizes  $e = 0$  of the error system (9) as follows:

**Theorem 1.** Let  $V(t, e)$  be an ISS-TCLF for the error system (9) and let  $\rho \in \mathcal{K}_\infty$  the corresponding function satisfying the condition (B2) of Definition 3. Then, the following time-varying state feedback  $\tilde{u} = \tilde{k}(t, e)$  input-to-state stabilizes  $e = 0$  of the error system (9).

$$\tilde{u} = \tilde{k}(t, e) := -p(t, e)L_{\tilde{g}}V^\top, \quad (14)$$

$$p(t, e) := \begin{cases} \frac{\omega + \sqrt{\omega^2 + \|L_{\tilde{g}}V\|^4}}{\|L_{\tilde{g}}V\|^2} & (L_{\tilde{g}}V \neq 0) \\ 0 & (L_{\tilde{g}}V = 0) \end{cases}, \quad (15)$$

$$\omega(t, e) := \frac{\partial V}{\partial t} + L_{\tilde{f}}V + \|L_{\tilde{h}}V\|\rho^{-1}(\|e\|). \quad (16)$$

**Remark 2** (Implementation of the controller (14)). *It should be mentioned that the state feedback (14) is given for the error system (9). Hence, to apply this feedback to the original system (4), the following input transformation is required:*

$$u = k(t, x) := \tilde{k}(t, x - x_d) + u_r(t). \quad (17)$$

### 3. PROBLEM FORMULATION

By using the rotation matrix  $R \in SO(3)$ , the rigid-body attitude dynamics is represented as follows:

$$\begin{aligned} \dot{R} &= RS(\omega) \\ \dot{\omega} &= J^{-1}(-\omega \times J\omega) + J^{-1}\tau + J^{-1}d, \end{aligned} \quad (18)$$

where  $\omega \in \mathbb{R}^3$  is the angular velocity,  $J \in \mathbb{R}^{3 \times 3}$  the symmetric positive-definite inertia matrix,  $\tau \in \mathbb{R}^3$  the input torque, and  $d \in \mathbb{R}^3$  the disturbance torque.

To consider the attitude tracking of system (18), we introduce the following assumption:

**Assumption 2.** *We suppose that the desired attitude trajectory  $R_d : [0, \infty) \rightarrow SO(3)$  is generated by the following system:*

$$\dot{R}_d = R_d S(\omega_d), \quad (19)$$

where  $\omega_d : [0, \infty) \rightarrow \mathbb{R}^3$  is the corresponding  $C^1$  desired angular velocity. We also suppose that both  $\omega_d$  and  $\dot{\omega}_d$  are bounded on  $[0, \infty)$ .

With the use of the error variables

$$\bar{R} = R_d^\top R, \quad \bar{\omega} = \omega - \bar{\omega}_d, \quad \bar{\omega}_d = \bar{R}^\top \omega_d, \quad (20)$$

it follows that the attitude error system is obtained as

$$\begin{aligned} \dot{\bar{R}} &= \bar{R}S(\bar{\omega}) \\ \dot{\bar{\omega}} &= J^{-1} [\Sigma(\bar{\omega}, \bar{\omega}_d)\bar{\omega} - S(\bar{\omega}_d)J\bar{\omega}_d - \bar{R}^\top \dot{\omega}_d] \\ &\quad + J^{-1}\tau + J^{-1}d, \end{aligned} \quad (21)$$

where

$$\Sigma(\bar{\omega}, \bar{\omega}_d) := S(J\bar{\omega}) + S(J\bar{\omega}_d) - [S(\bar{\omega}_d)J + JS(\bar{\omega}_d)]. \quad (22)$$

We then transform the error system (21) by using unit quaternions. Let  $q_d \in S^3$  be the desired quaternion satisfying  $R_d = \mathcal{R}(q_d)$ . The attitude tracking error, expressed in terms of unit quaternions, is defined as follows:

$$\bar{q} := (\bar{r}_0 \bar{r}^\top)^\top = q_d^{-1} \otimes q. \quad (23)$$

Based on the discussions of [12-14], we finally obtain the following error system:

$$\begin{aligned} \dot{\bar{q}} &= \frac{1}{2} \begin{bmatrix} -\bar{r}^\top \\ \bar{r}_0 I + S(\bar{r}) \end{bmatrix} \bar{\omega} \\ \dot{\bar{\omega}} &= J^{-1} [\Sigma(\bar{\omega}, \bar{\omega}_d)\bar{\omega} - \tau_{ff}(\bar{q}, \bar{\omega}_d, \dot{\omega}_d)] + J^{-1}(\tau + d), \end{aligned} \quad (24)$$

where  $\tau_{ff}$  is the feedforward torque defined as follows:

$$\tau_{ff}(\bar{q}, \bar{\omega}_d, \dot{\omega}_d) := J\mathcal{R}^\top(\bar{q}) + S(\bar{\omega}_d)J\bar{\omega}_d. \quad (25)$$

By using  $e := [\bar{r}^\top \bar{\omega}^\top]^\top$  and  $\tilde{u} := \tau - \tau_{ff}$ , the error system (24) can be transformed into the form of (9) with

$$\begin{aligned} \tilde{f}(t, e) &= \begin{bmatrix} -\frac{1}{2}\bar{r}^\top \bar{\omega} \\ -\frac{1}{2}[\bar{r}_0 I + S(\bar{r})]\bar{\omega} \\ J^{-1}\Sigma(\bar{\omega}, \bar{\omega}_d)\bar{\omega} \end{bmatrix}, \\ \tilde{g}(t, e) &= \tilde{h}(t, e) = \begin{bmatrix} O_{4 \times 3} \\ J^{-1} \end{bmatrix}. \end{aligned} \quad (26)$$

The objective of this paper is to achieve robust attitude tracking, i.e., input-to-state stabilizes  $e = ([-1, 0, 0, 0]^\top, 0)$  of the error system (24) based on the ISS-TCLF.

### 4. ROBUST TRACKING CONTROLLER DESIGN BASED ON ISS-TCLF

To apply the robust tracking controller, we design an ISS-TCLF for the error system (24). The following theorem is the first main result of this paper:

**Theorem 2.** *The following function is an ISS-TCLF for the error system (24):*

$$V(\bar{q}, \bar{\omega}) = 2\alpha(\bar{r}_0 + 1) + \frac{1}{2}(\bar{\omega} - \Gamma\bar{r})^\top J(\bar{\omega} - \Gamma\bar{r}), \quad (27)$$

where  $\alpha > 0$  is a positive constant and  $\Gamma \in \mathbb{R}^{3 \times 3}$  a constant positive definite symmetric matrix. Moreover, any class  $\mathcal{K}_\infty$  function  $\rho$  satisfies the condition (B2) of Definition 3.

#### Outline of the proof:

Here we give the outline of the proof of Theorem 2. As discussed in [11],  $V(\bar{q}, \bar{\omega})$  is a TCLF when  $d = 0$ . Hence, the condition (B1) of Definition 3 clearly holds. The condition (B2) is proved by a similar discussion in the proof of Theorem 2 of [10].

Based on (26) and (27), we can calculate  $L_{\tilde{f}}V$ ,  $L_{\tilde{g}}V$  and  $L_{\tilde{h}}V$  as follows:

$$\begin{aligned} L_{\tilde{f}}V &= -\alpha\bar{r}^\top \bar{\omega} \\ &\quad + (\bar{\omega} - \Gamma\bar{r})^\top \left[ \Sigma(\bar{\omega}, \bar{\omega}_d) - \frac{1}{2}J\Gamma(\bar{r}_0 I + S(\bar{r})) \right] \bar{\omega}, \\ L_{\tilde{g}}V &= L_{\tilde{h}}V = \frac{\partial V}{\partial e} \tilde{g}(t, e) = (\bar{\omega} - \Gamma\bar{r})^\top. \end{aligned} \quad (28)$$

Finally, we can construct the robust tracking controller by substituting (28) into (14) and designing the class  $\mathcal{K}_\infty$  function  $\rho^{-1}(\|e\|)$ .

### 5. SIMULATION STUDY

In this section, we confirm the effectiveness of the designed robust tracking controller on numerical simulations. The system and controller parameters are set to

$J = \text{diag}(3.0, 3.0, 3.0)$ ,  $\alpha = 1$ , and  $\Gamma = I$ . We employ the following function as  $\rho^{-1}(\|e\|)$ :

$$\rho^{-1}(\|e\|) = \beta(|\bar{r}_0 + 1| + \|\omega\|), \quad (29)$$

where  $\beta$  is a positive constant gain, and we set  $\beta = 0.5$  here.

In the following, for ease of understanding, we use the ZYX-Euler angles  $(\phi, \theta, \psi)$ [rad] to give the desired and initial attitudes. Note, however, that simulations are performed based on unit quaternions. Let us consider the following desired and initial attitudes:

$$\begin{bmatrix} \phi_d(t) \\ \theta_d(t) \\ \psi_d(t) \end{bmatrix} = \begin{bmatrix} \frac{\pi}{6} \sin\left(\frac{t}{2}\right) \\ -\frac{\pi}{6} \sin t \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \phi(0) \\ \theta(0) \\ \psi(0) \end{bmatrix} = \begin{bmatrix} \pi/3 \\ \pi/6 \\ \pi/4 \end{bmatrix}. \quad (30)$$

Moreover, the initial value of angular velocity is  $\omega(0) = [-\pi/9, \pi/6, -2\pi/9]^T$  [rad/s].

In the first simulation, we consider the following disturbance torque  $d_1(t)$ :

$$d_1(t) = [\sin(2t) \quad -1.5 \cos(2t) \quad 0]^T. \quad (31)$$

The simulation results are shown in Fig. 1. According to the figures, we can confirm that the trajectory tracking is achieved, subject to the disturbance in this case.

To clarify the problem of the controller (14), we then consider the following disturbance torque that contains constant terms:

$$d_2(t) = [\sin(2t) + 1.5 \quad -1.5 \cos(2t) - 1.0 \quad 0]^T. \quad (32)$$

Note that the difference between  $d_1(t)$  and  $d_2(t)$  is that the latter contains the constant terms, and the former does not. The simulation results with  $d_2(t)$  are depicted in Fig. 2. We can see that the effect of the disturbance is not mitigated. As a consequence, the trajectory tracking is not achieved.

These simulation results indicate that the controller (14) is not robust to the constant disturbances.

## 6. EXTENSION TO ROBUST ADAPTIVE TRACKING CONTROLLER

### 6.1. Controller design

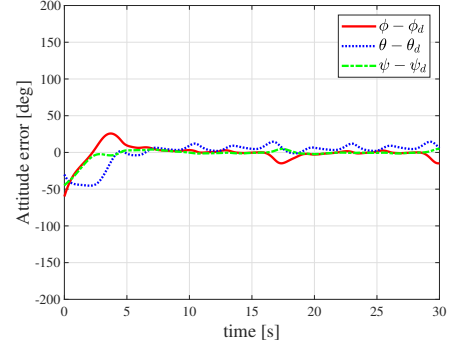
To deal with the problem pointed out in Section 5, we propose a robust adaptive tracking controller in this section. The key idea here is to represent the disturbance  $d(t)$  as

$$d(t) = d_{const} + \tilde{d}(t), \quad (33)$$

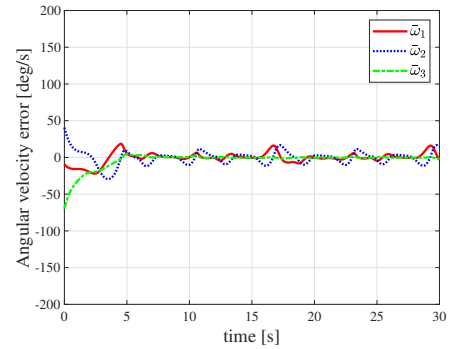
where  $d_{const}$  is the constant term and  $\tilde{d}(t)$  the residual time-varying term. To mitigate the effect of  $d_{const}$ , we introduce the additional state variable  $\hat{d}_{const}$ , which is the estimate of  $d_{const}$ . Based on the results of [9, 10, 14], we propose the following robust adaptive tracking controller:

$$\tilde{u} = \tilde{k}(t, e) + \hat{d}_{const}, \quad (34)$$

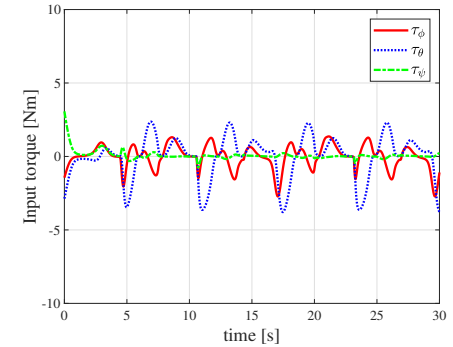
$$\dot{\hat{d}}_{const} = \Lambda L_{\tilde{g}} V^T, \quad (35)$$



(a) Attitude error



(b) Angular velocity error



(c) Input torque

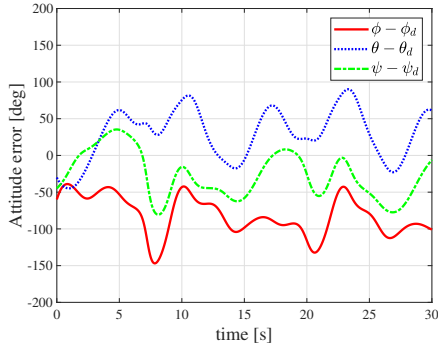
Fig. 1 Simulation results of the robust controller (14) with the disturbance  $d_1(t)$

where  $\tilde{k}(t, e)$  is the robust tracking controller designed in Section 4 and  $\Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ , where each  $\lambda_i > 0$  ( $i = 1, 2, 3$ ) is the adaptation gain for  $i$ -th element of  $\hat{d}_{const}$ .

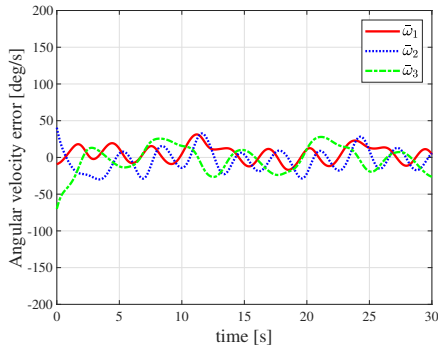
### 6.2. Simulation results

To confirm the effectiveness of the adaptive controller (34)–(35), we perform a computer simulation. We set the adaptive gain as  $\Lambda = \text{diag}(1, 1, 1)$ . Other parameters and simulation conditions are the same as those in Section 5.

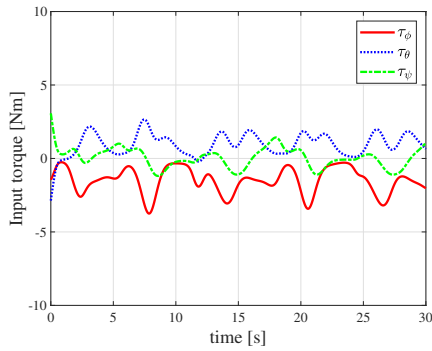
Simulation results with the disturbance  $d_1(t)$  are shown in Fig. 3. As can be seen in comparison with Fig. 1, introducing the adaptive compensation term  $\hat{d}_{const}$  does not deteriorate the control performance. Then we show the simulation results with the disturbance  $d_2(t)$  in Fig. 4. In comparison with Fig. 2, we can confirm that



(a) Attitude error



(b) Angular velocity error



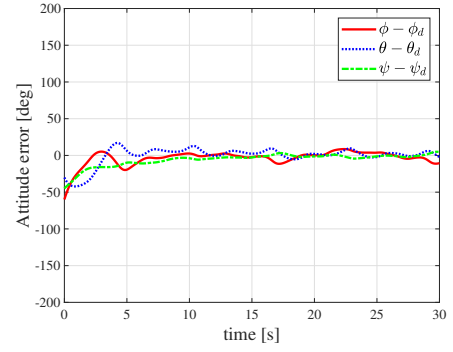
(c) Input torque

Fig. 2 Simulation results of the robust controller (14) with the disturbance  $d_2(t)$

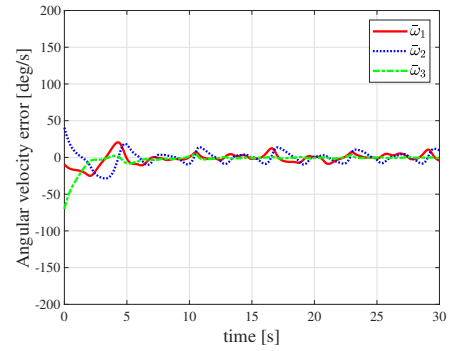
the trajectory tracking is achieved even under the disturbance  $d_2(t)$ . This indicates the effectiveness of the adaptive compensation of  $d_{const}$ .

## 7. CONCLUSIONS

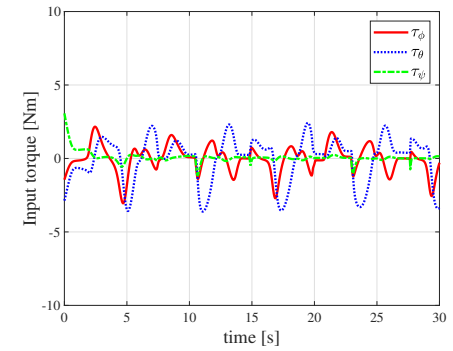
In this paper, we have considered the robust trajectory tracking problem of rigid body attitude. We first introduced the ISS-TCLF for the attitude error system and designed the robust trajectory tracking controller. Based on numerical simulation results, we found that the designed controller is not robust to disturbances containing constant terms. To solve this problem, we extended the controller to a robust adaptive tracking controller by introducing the constant disturbance estimator. The effectiveness of this adaptive controller is confirmed by the numerical simulation.



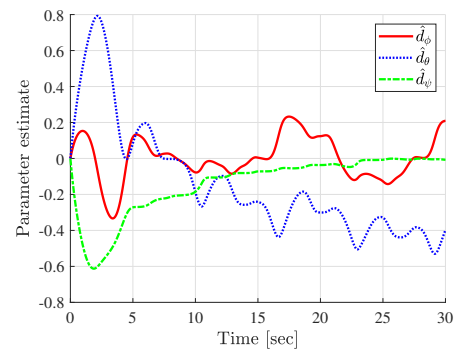
(a) Attitude error



(b) Angular velocity error



(c) Input torque



(d) Parameter estimate

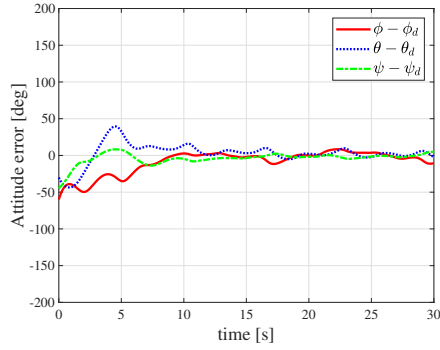
Fig. 3 Simulation results of the robust adaptive controller (34)–(35) with the disturbance  $d_1(t)$

Finally, future directions include a theoretical analysis of disturbance attenuation performance compared to non-adaptive tracking controllers. In addition, the extension to the global tracking controller is also a critical issue.

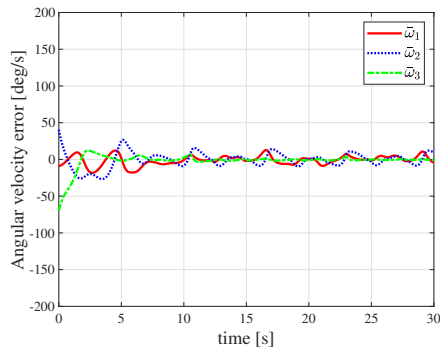
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## REFERENCES

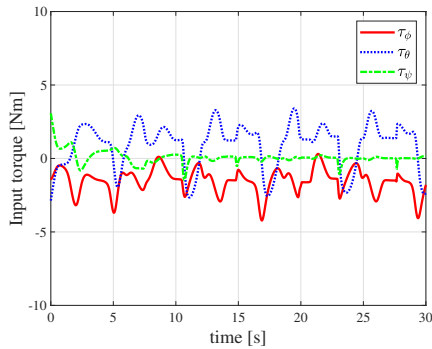
- [1] Q. Hu et al., “Adaptive Fixed-Time Attitude Tracking Control of Spacecraft With Uncertainty-Rejection Capability”, *IEEE Trans. Syst., Man, Cybern. Syst.*, Vol. 52, No. 7, pp. 4634–4647, 2022.
- [2] Z. Guo et al., “Global Finite-Time Stabilization of Spacecraft Attitude With Disturbances via Continuous Nonlinear Controller”, *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 59, No. 3, pp. 2608–2620, 2023.
- [3] H. A. Hashim and K. G. Vamvoudakis, “Adaptive Neural Network Stochastic-Filter-Based Controller for Attitude Tracking With Disturbance Rejection”, *IEEE Trans. Neural Netw. Learn. Syst.*, Vol. 35, No. 1, pp. 1217–1227, 2024.
- [4] C. Feng et al., “Adaptive Neural Network Stochastic-Filter-Based Controller for Attitude Tracking With Disturbance Rejection”, *IEEE Access*, Vol. 8, pp. 20013–20027, 2020.
- [5] J. Zhang et al., “Disturbance Observer-Based Adaptive Finite-Time Attitude Tracking Control for Rigid Spacecraft”, *IEEE Trans. Syst., Man, Cybern. Syst.*, Vol. 51, No. 11, pp. 6606–6613, 2021.
- [6] W. Zhu et al., “Disturbance Observer-Based Active Vibration Suppression and Attitude Control for Flexible Spacecraft”, *IEEE Trans. Syst., Man, Cybern. Syst.*, Vol. 52, No. 2, pp. 893–901, 2022.
- [7] T. Kanamori and H. Nakamura, “Trajectory Tracking Control for a Nonlinear System via a Time-Varying ISS-TCLF and Disturbance Observer”, *Proc. of CACS 2021*, pp. 1–6, 2021.
- [8] K. Chen and A. Astolfi, “Adaptive Control for Systems With Time-Varying Parameters”, *IEEE Trans. Autom. Control*, Vol. 66, No. 5, pp. 1986–2001, 2021.
- [9] Y. Satoh et al., “Robust Adaptive Trajectory Tracking of Nonlinear Systems Based on Input-to-state Stability Tracking Control Lyapunov Functions”, *IFAC-PapersOnLine*, Vol. 54, No. 14, pp. 388–393, 2021.
- [10] K. Ohno et al., “Disturbance Rejection Control of Rigid Body Attitude Based on Nonsmooth Control Lyapunov Function”, *Proc. of IEEE IECON 2018*, pp. 2293–2298, 2018.
- [11] S. Nomura et al., “Path-Following Control of Rigid Body Attitude by Using Minimum Projection Method”, *Proc. of IEEE CCTA 2018*, pp. 1591–1596, 2018.
- [12] C. G. Mayhew et al., “Quaternion-Based Hybrid Control for Robust Global Attitude Tracking”, *IEEE Trans. Autom. Control*, Vol. 56, No. 11, pp. 2555–2566, 2011.
- [13] B. Siciliano et al., *Robotics: Modelling, Planning and Control*, Springer, London, 2009.
- [14] T. Hatayama and H. Nakamura, “Discontinuous adaptive control of attitude of rigid body by using minimum projection method”, *Proc. of SICE 2014*, pp. 1424–1429, 2014.



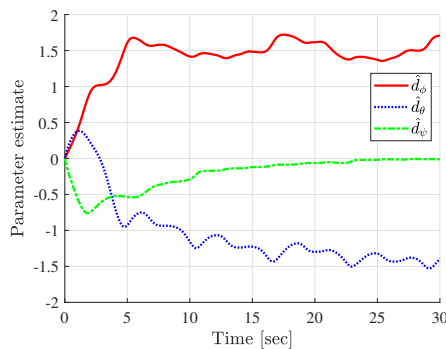
(a) Attitude error



(b) Angular velocity error



(c) Input torque



(d) Parameter estimate

Fig. 4 Simulation results of the robust adaptive controller (34)–(35) with the disturbance  $d_2(t)$

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