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Adaptive estimation of scheduling parameters using disturbance observers

Kotaro Kamewari[1†], Kana Shikada[2], Noboru Sebe[3], Masayuki Sato[4] and Ikuro Mizumoto[4]

[1]Department of Creative Informatics, Kyushu Institute of Technology, Fukuoka, Japan
(E-mail: kamewari.kotaro564@mail.kyutech.jp)
[2]Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Kyoto, Japan
(E-mail: shikada@kuaero.kyoto-u.ac.jp)
[3]Department of Intelligent and Control Systems, Kyushu Institute of Technology, Fukuoka, Japan
(E-mail: sebe@ics.kyutech.ac.jp)
[4]Faculty of Advanced Science and Technology, Kumamoto University, Kumamoto, Japan
(E-mail: sato.masayuki@mech.kumamoto-u.ac.jp, ikuro@gpo.kumamoto-u.ac.jp)

Abstract: In most cases, control systems are subject to uncertainties. Therefore, it is crucial to design robust controllers against such uncertainties. One representative approach for this purpose is gain-scheduled control. Gain-scheduled control is a powerful, robust control method that can be designed when uncertainties that are measurable online, known as scheduling parameters, exist. However, scheduling parameters are not always measurable online, so it is important to relax the condition of their availability. To address this, a method that estimates the scheduling parameter by a disturbance observer and the recursive least squares method was proposed. Nevertheless, a major drawback of the approach was that the estimate could converge to zero for some numerical examples. To overcome this drawback, this paper proposes a novel estimation method that combines a disturbance observer with an adaptive estimation mechanism. This paper also shows the effectiveness of the proposed method through a numerical example.

Keywords: disturbance observer, gain-scheduled control, H_∞ control, adaptive control, parameter estimation

1. INTRODUCTION

Scheduling parameters are parameters that cannot be determined in advance but can be measured online. When such parameters are available, they enable a control strategy that adjusts the control law according to the parameter's value. This approach is known as gain-scheduled control [1, 2] and allows the controller to adapt its behavior based on operating conditions dynamically. For example, in automotive engine control, real-time measurements such as vehicle speed and engine rotational speed can be used as scheduling parameters. By applying gain-scheduled control based on these variables, optimal fuel injection and ignition timing can be achieved for various driving situations such as low-speed driving, high-speed driving, acceleration, and deceleration. This enables smooth operation and high efficiency. In contrast, when using fixed controllers that do not vary with the scheduling parameters, the controller must be designed to account for all possible operating conditions in advance. As a result, the achievable control performance tends to be conservative and suboptimal in many situations. Therefore, gain-scheduled control is expected to yield superior control performance compared to fixed controllers, especially in systems with varying environments.

However, it is important to note that scheduling parameters are not always directly measurable in real-time. So it is necessary to relax the conditions of its availability. To address this limitation, several studies [3] have proposed gain-scheduled control with parameter estimation, where the scheduling parameter is estimated online and used to update the control law. This represents a promising approach to realizing more flexible and robust control systems. In [4], the estimation method using the disturbance observer and the recursive least squares method has been proposed. This method estimates the input and output sides of signals of the scheduling parameter by the disturbance observer, and then applies the recursive least squares method. However, using the method in [4], the estimated values converged to zero for some numerical examples.

To address this issue, this paper proposed a method for estimating scheduling parameters using a combination of a disturbance observer and an adaptive estimation mechanism. Similar to the method in [4], a disturbance observer estimates the input and output signals of scheduling parameters, and the scheduling parameters are estimated by applying an adaptive estimation mechanism to those variables. The effectiveness of the gain-scheduled control based on the estimated values is confirmed by comparing its control performance with that of the gain-scheduled control using the actual scheduling parameters.

Notations : The identity matrix and the zero matrix are denoted by I and O, respectively. In general, signals are functions of time t. However, for the sake of simplicity, we do not explicitly describe the function t.

2. PROBLEM SETTINGS

A control algorithm using varying controllers according to scheduling parameters is called a gain-scheduled control. The block diagrams representing conventional robust control and gain-scheduling control are shown in Figs. 1 and 2. As shown in Fig. 1, the controller is independent of the scheduling parameters in conven-

† Kotaro Kamewari is the presenter of this paper.

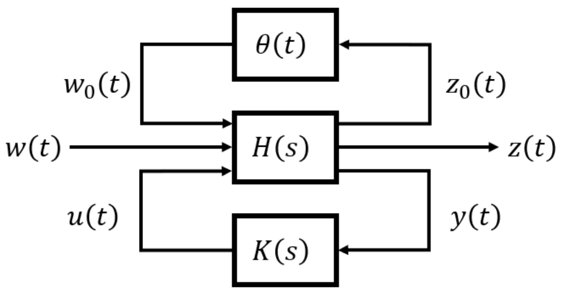

Fig. 1 Block diagram of conventional robust control

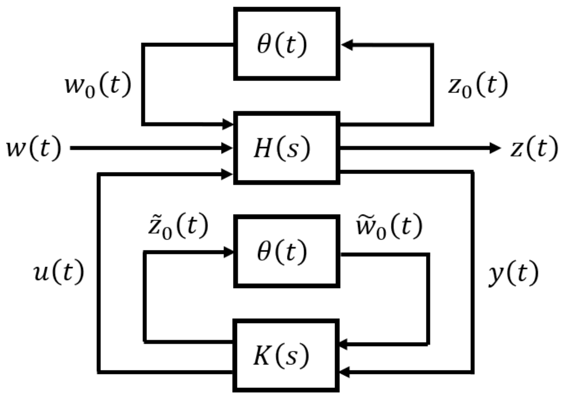

Fig. 2 Block diagram of gain-scheduled control

tional robust control, whereas the gain-scheduled controller changes depending on the scheduling parameters as shown in Fig. 2. The gain-scheduled controller is expected to achieve better performance than the conventional fixed controller of robust control. In this paper, we will consider a particular case where the scheduling parameter $\theta(t)$ is a scalar value and cannot be measured online. Assume that the state space representation of the generalized plant $H(s)$ shown in Fig. 2 is given by the following equations.

$$\dot{x} = Ax + B_0 w_0 + B_1 w + B_2 u, \tag{1}$$

$$z_0 = C_0 x + D_{00} w_0 + D_{01} w + D_{02} u, \tag{2}$$

$$z = C_1 x + D_{10} w_0 + D_{11} w + D_{12} u, \tag{3}$$

$$y = C_2 x + D_{20} w_0 + D_{21} w, \tag{4}$$

where x is the state variable, u is the control input, y is the observed output, w is the reference input, and z is the evaluated output. Furthermore, also assume that the relation between the scheduling parameter $\theta(t)$ and the signals w_0 and z_0 is given by

$$w_0 = \theta(t) z_0. \tag{5}$$

Note that u and y are the only known signals.

3. PROPOSED METHOD

This paper aims to estimate scheduling parameters using a combination of a disturbance observer and an adaptive estimation mechanism.

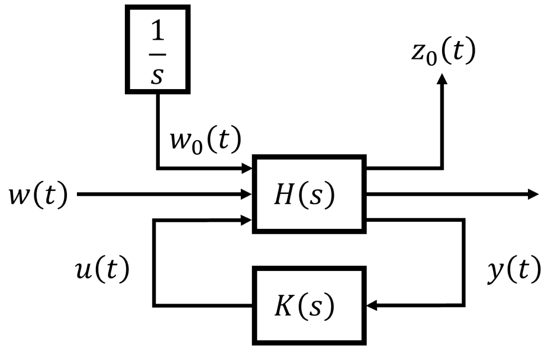

Fig. 3 Block diagram with $w_0(t)$ considered as a disturbance

Using a disturbance observer, the signals w_0 and z_0 can be estimated [4]. Next, the scheduling parameter θ is estimated through an adaptive estimation mechanism that utilizes the estimated values of $\hat{w}_0$ and $\hat{z}_0$. In this process, the convergence rate of the estimated scheduling parameter $\hat{\theta}$ can be modified appropriately by tuning the design parameter, the forgetting factor λ.

3.1. Estimation of w_0 and z_0 by disturbance observer [4]

The disturbance observer is a system that estimates the disturbances added to the nominal plant [5, 6]. The disturbance observer proposed in [5, 6] estimates the state of the augmented system composed of the nominal plant and disturbance-generating models. This paper regards the output signal w_0 of the scheduling parameter block as a disturbance and estimates it using a disturbance observer. The block diagram is shown in Fig. 3. It is assumed that the disturbance w_0 is generated by a linear dynamical system, whose state-space representation is given by

$$\dot{x}_d = A_d x_d, \tag{6}$$

$$w_0 = C_d x_d, \tag{7}$$

where x_d is the state variable and A_d and C_d are the coefficient matrices. In this paper, the disturbance is assumed to be a constant signal, which implies that $A_d = O$ and $C_d = I$. Based on this assumption, the state-space representation of the augmented system, composed of the given plant and the disturbance-generating model, is formulated as follows.

$$\begin{bmatrix} \dot{x} \\ \dot{x}_d \\ \hline z_0 \\ z \\ y \end{bmatrix} = \left[\begin{array}{cc|cc} A & B_0 C_d & O & B_2 \\ O & A_d & O & O \\ \hline C_0 & D_{00} C_d & D_{01} & D_{02} \\ C_1 & D_{10} C_d & D_{11} & D_{12} \\ C_2 & D_{20} C_d & D_{21} & O \end{array}\right] \begin{bmatrix} x \\ x_d \\ \hline w \\ u \end{bmatrix}. \tag{8}$$

If this augmented system (8) is observable, the observer gain matrix $\bar{H} = [H_1^{\mathrm{T}} \; H_2^{\mathrm{T}}]^{\mathrm{T}}$ can be designed, and a disturbance observer for the augmented system is obtained as follows.

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}}_d \end{bmatrix} = \begin{bmatrix} A & B_0 C_d \\ O & A_d \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{x}_d \end{bmatrix} + \begin{bmatrix} B_2 \\ O \end{bmatrix} u + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} (y - \hat{y}), \quad (9)$$

$$\hat{y} = \begin{bmatrix} C_2 & D_{20} C_d \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{x}_d \end{bmatrix}, \quad (10)$$

where $\hat{x}, \hat{x}_d, \hat{y}$ are estimates. With the estimated values $\hat{x}$ and $\hat{x}_d$, we can obtain the estimated values $\hat{w}_0$ and $\hat{z}_0$ as follows.

$$\hat{z}_0 = C_0 \hat{x} + D_{00} C_d \hat{x}_d, \quad (11)$$

$$\hat{w}_0 = C_d \hat{x}_d. \quad (12)$$

Remark 1: From the PBH rank test [7], the condition for the augmented system (8) to be observable is that the matrix

$$\begin{bmatrix} A - \mu I & B_0 C_d \\ O & A_d - \mu I \\ C_2 & D_{20} C_d \end{bmatrix}. \quad (13)$$

is the column full-rank for all the eigenvalues μ of the augmented system. Considering that $A_d = O$, the column rank of the matrix shown in (13) cannot be larger than the sum of the numbers of the state x and the measurement output y. In other words, the estimation method using disturbance observers proposed in this paper cannot estimate the number of scheduling parameters beyond the dimension of the measurement output y.

3.2. Estimation of the scheduling parameter $\boldsymbol{\theta}$

In principle, the scheduling parameters can be estimated by computing the ratio of the estimated values derived from (5). However, direct division may not always yield accurate estimates due to the presence of outliers, the influence of noise, and the potential for division by zero. In addition, the estimation method proposed in [4] had a drawback. Because of the regularization factor, which prevents the zero division, the estimated value converged to zero for some uncertain parameters. Therefore, this paper estimates the estimate $\hat{\theta}$ of the scheduling parameter θ by applying an adaptive estimation mechanism [8]. The dynamics of $\hat{\theta}$ is obtained as the following differential equation.

$$\dot{\hat{\theta}} = -\Gamma \hat{z}_0 (\hat{\theta}^{\mathrm{T}} \hat{z}_0 - \hat{w}_0), \quad (14)$$

where $\Gamma = \Gamma^{\mathrm{T}}\ (\succ O)$ is a adaptive gain. The convergence rate to the solution can be adjusted by tuning this value. The estimated value $\hat{\theta}$ of the scheduling parameter can be obtained by solving this differential equation.

However, there exists a significant issue. Specifically, adjusting the adaptive gain Γ affects not only the convergence rate but also the final value of the estimation. As a result, the gain must be tuned through trial and error based on simulation results during the design phase, which is impractical for real-world applications. To address this issue, the recursive weighted least squares method is employed to dynamically determine the adaptive gain Γ. Let the time-varying adaptive gain be $\Gamma(t)$. Then, the updating rule for $\Gamma(t)$ can be given as the following differential equation.

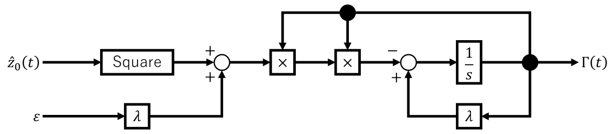

Fig. 4 Block diagram related to $\Gamma(t)$ and $\hat{z}_0$

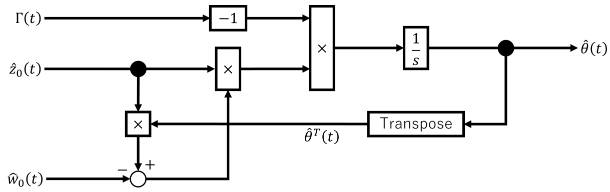

Fig. 5 Estimation flow of scheduling parameter

$$\dot{\Gamma}(t) = \lambda \Gamma(t) - \Gamma(t) \big(\hat{z}_0 \, \hat{z}_0^{\mathrm{T}} + \lambda \varepsilon I \big) \Gamma(t), \quad (15)$$

where λ is a forgetting factor and ε is a regularization factor. The block diagram of this differential equation is shown in Fig. 4. As a result, the adaptive gain can be automatically obtained without relying on trial-and-error through simulation. Furthermore, the convergence rate of the estimated value $\hat{\theta}$ can be adjusted by tuning the forgetting factor λ. Based on the above, equations (14) and (15) can be combined, and the estimation mechanism can be expressed as follows.

$$\dot{\hat{\theta}} = -\Gamma(t) \, \hat{z}_0 (\hat{\theta}^{\mathrm{T}} \hat{z}_0 - \hat{w}_0). \quad (16)$$

The block diagram of this mechanism is shown in Fig. 5. This enables estimation of the scheduling parameter without the estimate converging to zero, while allowing the convergence rate to be adjusted solely through the design parameter.

4. NUMERICAL EXAMPLE

In this section, the effectiveness of the proposed method is demonstrated through a numerical example. First, it is verified that the scheduling parameter can be accurately estimated using the proposed method, and the result is compared with the estimation obtained by the method proposed in [4]. Second, the effect of the forgetting factor on the convergence rate is examined by varying its value. Finally, we check the response to the step input for two types of control: gain-scheduled control with estimated scheduling parameters and gain-scheduled control with actual scheduling parameters.

4.1. Problem setting

4.1.1. Controlled object

Let us consider the double cart system shown in Fig. 6 as a plant for this numerical example, where m_1, m_2 are

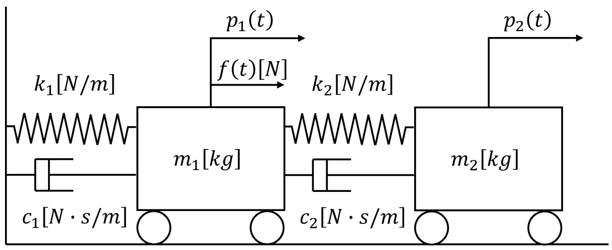

Fig. 6 Double cart system

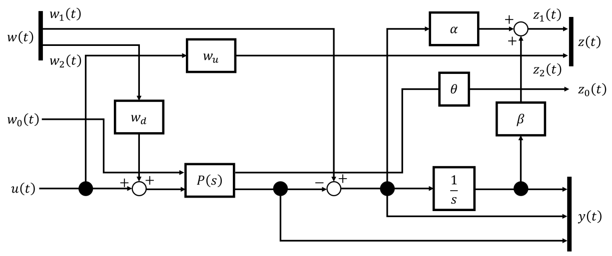

Fig. 7 Generalized plant for designing gain-scheduled and fixed robust controllers

masses, k_1, k_2 are spring constants, and c_1, c_2 are damping constants. The $p_1(t)$, $p_2(t)$ represent the position of each cart from its initial balanced position. The $f(t)$ is the force generated by the drive motor. The equation of motion is given by

$$\begin{cases} m_1\ddot{p}_1 = k_2(p_2 - p_1) + c_2(\dot{p}_2 - \dot{p}_1) \\ \qquad\quad -k_1 p_1 - c_1\dot{p}_1 + f, \\ m_2\ddot{p}_2 = k_2(p_1 - p_2) + c_2(\dot{p}_1 - \dot{p}_2). \end{cases} \tag{17}$$

The parameters used in the numerical experiments were set as follows.

$$\begin{cases} m_1 = 10, \quad m_2 = 20, \\ k_1 = 10, \quad k_2 = 5, \\ c_1 = 10, \quad c_2 = 20. \end{cases}$$

Among the physical parameters, suppose that there is an uncertainty in the magnitude of m_1, specifically, that m_1 is additively perturbed by θ. This parameter θ is treated as a scheduling parameter. In this case, the values of the scheduling parameters are changed to $\theta = 0$, ± 0.5, ± 1.

4.1.2. Gain-scheduled controller design

For this system, a two-degree-of-freedom (2-DOF) servo controller is designed based on H_∞ control theory. The generalized plant used for the synthesis is shown in Fig. 7. The transfer function from $w_1(t)$ to $z_1(t)$ characterizes the tracking performance of the servo system through the weighting function $W_s(s) = \frac{\alpha s+\beta}{s}$. The transfer function from $w_1(t)$ to $z_2(t)$ is introduced to evaluate and attenuate the magnitude of the control input. Furthermore, the transfer function from $w_2(t)$ to $z_1(t)$ is incorporated to prevent pole-zero cancelation between the plant and the controller. For this generalized plant, the following weighting functions are set to design a gain-scheduled controller.

$$\begin{cases} \alpha = 0.95, \quad \beta = 0.5, \\ w_u = 0.1, \quad w_d = 0.1. \end{cases}$$

4.1.3. Settings for scheduling parameter estimation

The disturbance observer is designed to estimate the scheduling parameter using the pole assignment method. The desired pole placement is specified as

$$\{-2, -1, -0.5, -1.5, -2.75\},$$

and the resulting observer gain is given by

$$[\, 12.6731 \quad 3.75 \quad 2.2692 \quad 4.75 \quad -165 \,]^{\mathrm{T}}.$$

About the parameters on estimation, the forgetting factor λ in the proposed method, which affects the convergence time of the estimated value, is set to $\lambda = 2, 10$. And the regularization factor, which prevents $\Gamma(t)$ from going to infinity, is set to $\varepsilon = 1 \times 10^{-3}$. The initial value of $\Gamma(t)$ is set to $\Gamma(0) = 1$. For the previous method [4], the time constant T for the first-order low-pass filter is set to $T = 0.9$, and the regularization factor ε as $\varepsilon = 1 \times 10^{-4}$.

4.2. Experimental results

4.2.1. Estimation of scheduling parameters

The results of the estimation of the scheduling parameters are shown in Figs. 8 to 10. Figures 8 and 9 show the estimated value of the scheduling parameter using the proposed method in this paper. Figure 10 shows the estimated value of the scheduling parameter using the proposed method in [4]. Unlike in Fig. 10, it can be confirmed that the estimation does not converge to zero, and a valid estimation is obtained. In addition, it is observed that the convergence time to the estimated value varies depending on the value of the forgetting factor λ. The time responses of the adaptive gain $\Gamma(t)$ with $\lambda = 2, 10$ are shown in Figs. 11 and 12.

4.2.2. Gain-scheduled control using estimated values

The input and output responses to the unit step input for the gain-scheduled control with estimated values and the gain-scheduled control with actual scheduling parameters are shown in Figs. 13 to 16.

Figures 13 and 14 show the output of the closed-loop systems, and Figs. 15 and 16 show the control inputs of the closed-loop systems, respectively. The signals with the gain-scheduled controller with the estimated parameter are shown in Figs. 13 and 15. Figures. 14 and 16 show the signals with the gain-scheduled controller with the actual scheduling parameter.

It is confirmed that gain-scheduled control can be achieved using the estimated values. Furthermore, no significant degradation in control performance is observed compared to the case in which the actual scheduling parameters are available.

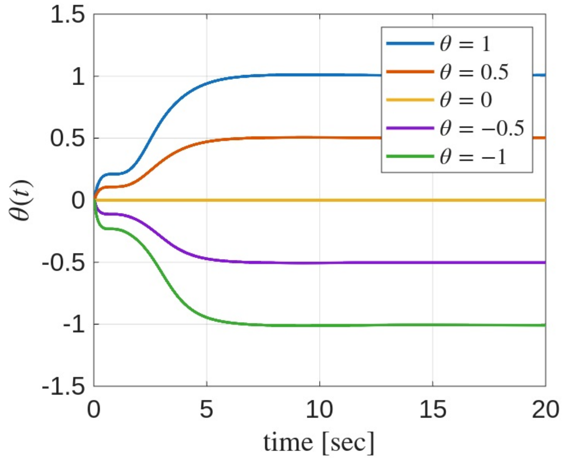

Fig. 8 Estimated value of scheduling parameter with proposed method ($\lambda = 2$)

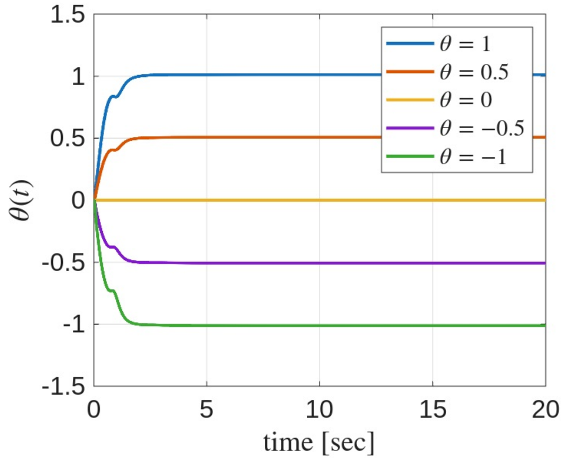

Fig. 9 Estimated value of scheduling parameter with proposed method ($\lambda = 10$)

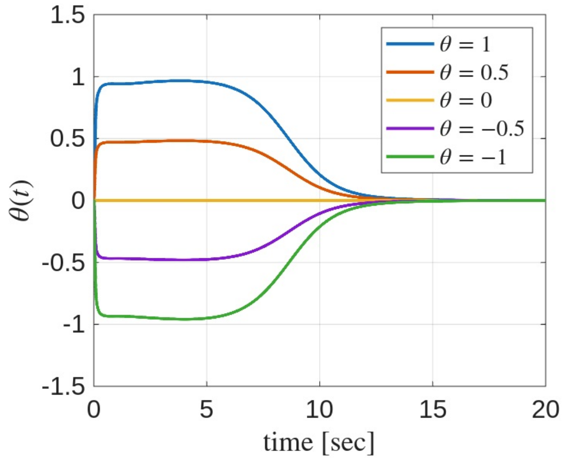

Fig. 10 Estimated value of scheduling parameter with the previous method [4]

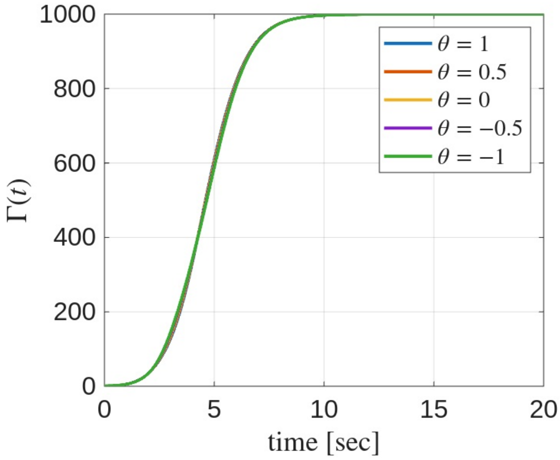

Fig. 11 Time response of adaptive gain $\Gamma(t)$ ($\lambda = 2$)

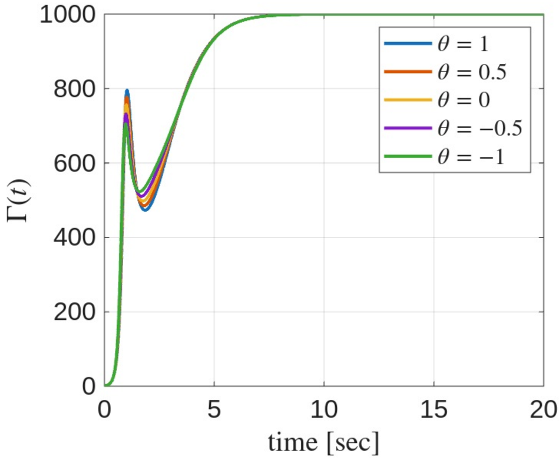

Fig. 12 Time response of adaptive gain $\Gamma(t)$ ($\lambda = 10$)

5. CONCLUSIONS

This paper proposes to estimate the scheduling parameters using a combination of a disturbance observer and an adaptive estimation mechanism. The effectiveness of the proposed method is demonstrated through a numerical example. Future work includes a theoretical analysis of the extent to which control performance degrades when using estimated scheduling parameters, as well as an investigation into optimal gain-scheduled controller design based on such estimates [9-12].

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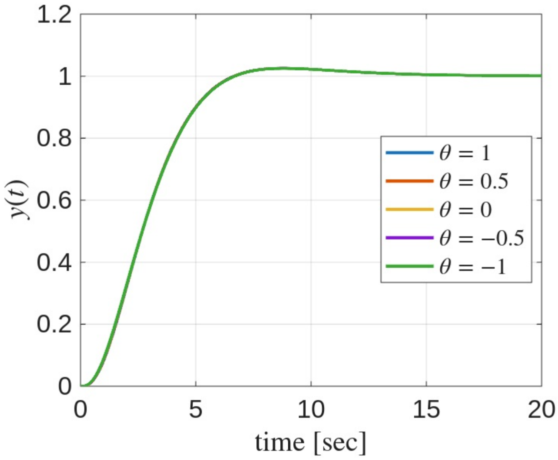

Fig. 13 Step responses of control output achieved by the gain-scheduling controller with estimated parameter ($\lambda = 10$)

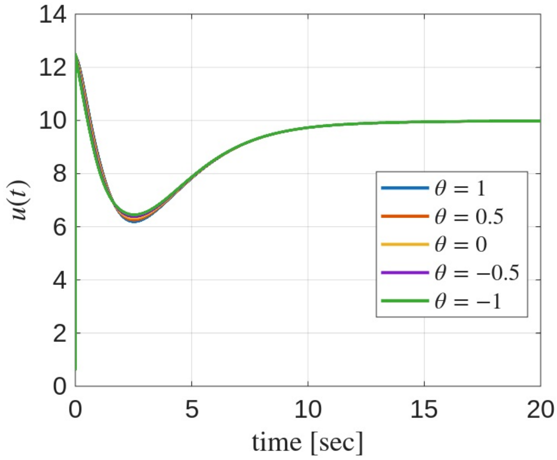

Fig. 15 Step responses of control input achieved by the gain-scheduling controller with estimated parameter ($\lambda = 10$)

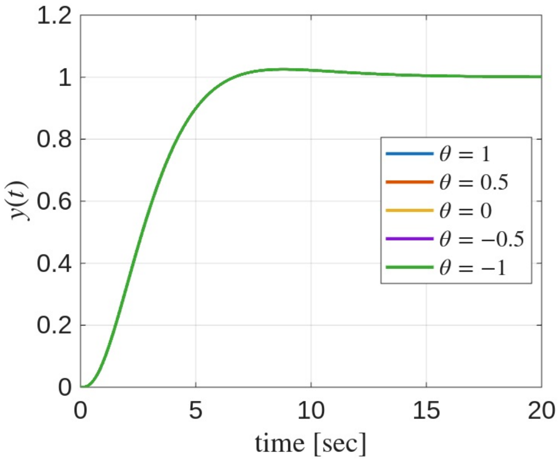

Fig. 14 Step responses of control output achieved by the gain-scheduling controller with actual parameter

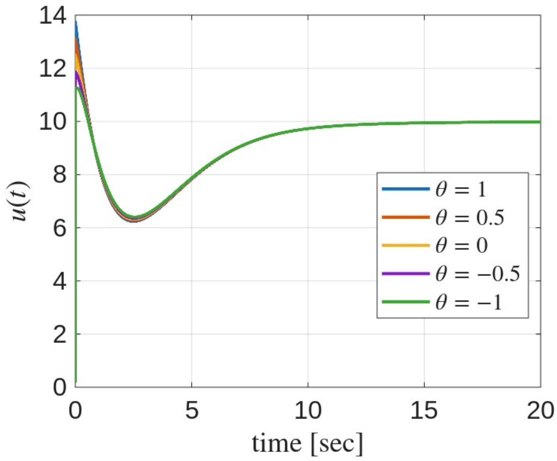

Fig. 16 Step responses of control input achieved by the gain-scheduling controller with actual parameter

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