

Plant Input Mapping Discretization for Strictly Proper Feedback System in State Space Form

Shuya Sawahata^{1†} and Keisuke Yagi²

¹Graduate School of Science and Engineering, Ibaraki University, Hitachi, Japan

²Domain of Mechanical Systems Engineering, Ibaraki University, Hitachi, Japan
(Tel : +81-294-38-5196; E-mail: keisuke.yagi.dc@vc.ibaraki.ac.jp)

Abstract: This paper is concerned with the discretization of a feedback control system capable of preserving the stability of the underlying system for any non-pathological sampling interval. A class of techniques achieving such discretization, called the plant input mapping method, has been developed in our previous studies. An issue remains in the method that it is applicable only to a bi-proper system. The present study is aimed at extending our discretization method to a strictly proper system. An academic example illustrating the features and an experiment on digital implementation are provided.

Keywords: Discretization, Linear feedback system, State space form, Transmission zeros, Strictly proper systems

1. INTRODUCTION

Because of the remarkable progress of computer technology, the control system has in general been implemented digitally, and a variety of applications even the electric and electronic systems take advantage of digital implementation. Design of a digital controller should consider two kinds of the time domain, the continuous-time (CT) for the plant and discrete-time (DT) for the control system. There exist strong approaches for a unified treatment of the time domains under a preliminary determined sampling interval, such as the lifting technique [1] and variations of DT operators [2], [3]. However, digital redesign approaches are still demanded because the process of controller design can be completed in the CT domain through well-established designing techniques without consideration for the sampling interval. Also, digital redesign is aimed at reproducing the control performance of the underlying system precisely. This aspect provides an effective solution to the demand of digitalization of an existing system installing an analog controller without changing its control characteristics.

Plant input mapping (PIM) method [4] is one of the digital redesign methods, and its discretization focuses on the plant input signal of a given CT feedback system rather than each system block. The plant input signal always includes the zeros (transmission zeros) exactly corresponding to the plant poles and the poles exactly corresponding to the closed loop pole [5]. The PIM method employs the matched-pole-zero (MPZ) model [6] for discretization of the plant input signal and determines the parameters of a prescribed digital controller to achieve the discretized plant input signal. This discretization process allows us to derive a digital form of the underlying feedback system with preserving its stability for any non-pathological sampling interval.

Based on this remarkable advantage, feedback controllers for electric [7] and electronic [8] systems achieved stable digital implementation despite a comparatively large sampling interval. On the other hand, the PIM method was first applicable only to the single-input single-output system described in the transfer function form. This was due to the absence of the MPZ model in the state space form, where the zeros cannot be

dealt with explicitly like the poles. This issue has been solved, and some algorithms for the MPZ discretization in the state space have been presented [9], [10], [11], succeeding in the extension of the PIM discretization to the state space form [12]. However, there exists a condition for the closed-loop discretization that the feedback controller must have a direct term represented by a non-singular matrix; that is, the controller must be bi-proper. This condition disrupts the PIM discretization of a feedback system including, e.g., an integrator, a simple lowpass filter, etc, which are frequently used in practical applications. Although some solutions were discussed in our previous study [13], those approaches cannot ensure the stability because the closed loop poles of the PIM digital controller do not match exactly with those of the MPZ model of the underlying system.

This paper proposes a modified PIM discretization algorithm applicable to a strictly proper system as well as the bi-proper system. The proposed algorithm employs a variation of the MPZ models called the mapping zero model, which considers only the characteristics of the zeros and does not those of the poles. The prescribed digital controller is first determined by the mapping zero model, and it is second modified to achieve the desired pole placement of the closed loop system. This process reorders one part of the conventional algorithm, where the parameters are first modified in the MPZ discretization algorithm, and then the digital controller is determined through parameter matching. Since the mapping algorithm is for a single block discretization, the reorder of the process brings an essential difference in that the parameter modification step can take the feedback structure into account. The effectiveness of the proposed algorithm is illustrated by an academic example. Also, an experiment on digital implementation of a current regulator for a five-phase stepping motor is carried out with a comparatively large sampling interval regarding the system response.

2. PRELIMINARIES

2.1 System descriptions

This paper deals with the CT and DT systems in the state space form, and all the systems are assumed to be minimal realization. The CT system is described by

$$\begin{aligned}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}\bar{u}(t), \\ \bar{y}(t) &= \bar{C}\bar{x}(t) + \bar{D}\bar{u}(t),\end{aligned}\quad (1)$$

where $\bar{u}(t), \bar{y}(t) \in \mathbb{R}^\ell$ are the input and output signals, and the matrices have the corresponding size to them. The upper bar indicates that the signals and systems belong to the CT domain. The DT system is described by

$$\begin{aligned}\delta x_k &= Ax_k + Bu_k, \\ y_k &= Cx_k + Du_k,\end{aligned}\quad (2)$$

where the subscript k is a positive integer, which indicates the time with the sampling interval T as $t = kT$. The operator δ is the delta operator [2], which is defined using the shift operator q and the sampling interval T by

$$\delta = \frac{q-1}{T} \quad (3)$$

The state variables and parameter matrices frequently have a subscript for indicating their belonging systems, for instance, x_p for the plant and A_c for the compensator.

2.2 Discrete time models

This paper deals with three kinds of the DT model for discretization of a CT system, namely the step-invariant model [14], the mapping zero model, and the MPZ model [10], [11], and this subsection presents a brief description of them. All the DT models suppose that the zero-order-hold and sampler are employed as the D/A and A/D converter respectively, and all the initial condition is set to 0.

The step-invariant model is derived from the underlying CT system by the following mapping law:

$$\begin{aligned}A_{SIM} &= \frac{e^{\bar{A}T} - I}{T}, \quad B_{SIM} = \frac{1}{T} \int_0^T e^{\bar{A}\tau} d\tau \bar{B}, \\ C_{SIM} &= \bar{C}, \quad D_{SIM} = \bar{D}.\end{aligned}\quad (4)$$

The step-invariant model can preserve the controllability and observability of the underlying system for any non-pathological sampling interval [15].

When the direct term is described by a non-singular matrix, the mapping zero model is given by

$$\begin{aligned}A_{MZ} &= \frac{e^{(\bar{A} - \bar{B}\bar{D}^{-1}\bar{C})T} - I}{T} + \frac{1}{T} \int_0^T e^{(\bar{A} - \bar{B}\bar{D}^{-1}\bar{C})\tau} d\tau (\bar{B}\bar{D}^{-1}\bar{C}), \\ B_{MZ} &= \frac{1}{T} \int_0^T e^{(\bar{A} - \bar{B}\bar{D}^{-1}\bar{C})\tau} d\tau \bar{B}, \quad C_{MZ} = \bar{C}, \quad D_{MZ} = \bar{D}.\end{aligned}\quad (5)$$

The mapping zero model contains the transmission zeros mapped according to the same law as the poles of the step-invariant model, while the mapping law for the poles is different. Controllability and observability are preserved as well as the step-invariant model. Note that the non-singular direct term is not a mandatory condition to derive the mapping zero model (See [11]).

The MPZ model is given by

$$A_{MPZ} = A_{MZ} - \frac{1}{T} \int_0^T e^{(\bar{A} - \bar{B}\bar{D}^{-1}\bar{C})\tau} d\tau (\bar{B}\xi KW^{-1}), \quad (6)$$

$$B_{MPZ} = B_{MZ}, \quad C_{MPZ} = C_{MZ} - \bar{D}\xi KW^{-1}, \quad D_{MPZ} = D_{MZ}.$$

where $\xi = \mathbf{1} \in \mathbb{R}^\ell$, K is the internal state feedback gain to achieve the pole placements in accordance with the mapping rule, and W is the similarity transformation matrix for the canonical form. As indicated by this

formulation, this model is obtained by modification of the pole-placement of the mapping zero model. The poles and transmission zeros of the underlying system are mapped into the DT domain by the same law, and the controllability and observability are preserved as well.

2.3 Plant Input Mapping method

The plant input mapping method is a global discretization technique and is aimed at preserving the characteristics of the underlying closed-loop system rather than a single system block. With reference to Fig. 1, the discretization process is explained.

Suppose that the underlying CT feedback system has already been designed to be minimal and stable, and the direct term of the compensator is described by a non-singular matrix. Then, the plant and the feedback system are described by

$$\begin{aligned}\dot{\bar{x}}_p(t) &= \bar{A}_p \bar{x}_p(t) + \bar{B}_p \bar{u}(t) \\ \bar{y}(t) &= \bar{C}_p \bar{x}_p(t)\end{aligned}\quad (7)$$

and

$$\begin{aligned}\dot{\bar{x}}_c(t) &= -\bar{A}_{c1} \bar{x}_p(t) + \bar{A}_{c2} \bar{x}_c(t) + \bar{B}_c \bar{r}(t) \\ \bar{u}(t) &= -\bar{C}_{c1} \bar{x}_p(t) + \bar{C}_{c2} \bar{x}_c(t) + \bar{D}_c \bar{r}(t)\end{aligned}\quad (8)$$

where $x_p \in \mathbb{R}^n$ and $x_c \in \mathbb{R}^m$ denote the state vector of the plant and the compensator, respectively.

To focus on the characteristics of the plant input signal, a virtual system called the plant input state equation (PISE) is defined. The PISE describes the relationship between the reference input signal and the plant input signal in a feedback loop, which is obtained by the state equation of the closed-loop system and output equation of the compensator as

$$\begin{aligned}\begin{bmatrix} \dot{\bar{x}}_p(t) \\ \dot{\bar{x}}_c(t) \end{bmatrix} &= \begin{bmatrix} \bar{A}_p - \bar{B}_p \bar{C}_{c1} & \bar{B}_p \bar{C}_{c2} \\ -\bar{A}_{c1} & \bar{A}_{c2} \end{bmatrix} \begin{bmatrix} \bar{x}_p(t) \\ \bar{x}_c(t) \end{bmatrix} + \begin{bmatrix} \bar{B}_p \bar{D}_c \\ \bar{B}_c \end{bmatrix} \bar{r}(t) \\ \bar{u}(t) &= \begin{bmatrix} -\bar{C}_{c1} & \bar{C}_{c2} \end{bmatrix} \begin{bmatrix} \bar{x}_p(t) \\ \bar{x}_c(t) \end{bmatrix} + \bar{D}_c \bar{r}(t)\end{aligned}\quad (9)$$

According to the control zero principle [5], the transmission zeros of the PISE always eliminate the original poles of the plant. Also, the PISE obviously has the same poles as the whole feedback system.

The PISE is then discretized by the MPZ model, which is described by

$$\begin{aligned}\delta \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} &= \begin{bmatrix} A_{MPZ,11} & A_{MPZ,12} \\ A_{MPZ,21} & A_{MPZ,22} \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + \begin{bmatrix} B_{MPZ,1} \\ B_{MPZ,2} \end{bmatrix} \Gamma r_k \\ u_k &= \begin{bmatrix} C_{MPZ,1} & C_{MPZ,2} \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + D_{MPZ} \Gamma r_k\end{aligned}\quad (10)$$

On the other hand, the prescribed form of the PIM digital controller has the PISE form given by

$$\begin{aligned}\delta \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} &= \begin{bmatrix} A_{SIM} - B_{SIM} K_1 & B_{SIM} C_c \\ -K_2 & A_c \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + \begin{bmatrix} B_{SIM} D_c \\ B_c \end{bmatrix} \Gamma r_k, \\ u_k &= [-K_1 \quad C_c] \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + D_c \Gamma r_k,\end{aligned}\quad (11)$$

where the plant is discretized by the step-invariant model. The parameters of the DT compensator are to be determined together with the state feedback gain K_1 and K_2 to match the MPZ model of the CT PISE by

$$A_C = A_{MPZ,22}, B_C = B_{MPZ,2}, C_C = C_{MPZ,2}, D_C = D_{MPZ}, \quad (12)$$

$$K_1 = -C_{MPZ,1}, K_2 = -A_{MPZ,21}.$$

The constant gain Γ is determined by the closed-loop dc gain of the underlying CT system and the discretized system as follows:

$$\Gamma = \Gamma_{DT}^{-1} \Gamma_{CT} \quad (13)$$

When the dc gain is singular due to the existence of the integrator pole, the unit matrix is available for Γ . Finally, the DT observer is designed to be stable, where an arbitrary method is available.

2.4 Problems of the PIM method

While the PIM method can provide a digital form of the CT feedback controller with preserving its stability irrespective of the sampling interval, the discretization process supposes that the direct term of the compensator is represented by a non-singular matrix. That is, the compensator must be a bi-proper system. This assumption is essential in the parameter matching phase described by eqns. (10) and (11); when D_C is singular, the parameters determined by eqn. (12) cannot match eqn. (11) with eqn. (10). This unmatching causes the deviation of the pole placement from the desired one, and then, the stability of the discretized controller is not guaranteed.

To address this issue, adding a non-singular direct term to the underlying system was considered in [4]. This approach enables the parameter matching without affecting the control zero principle and the original pole placement of the underlying system. However, the number and placement of the zeros of the compensator are varied. The additional direct term should be determined in a manner of trial and error with taking the sampling interval into account, and thus, it is not an effective solution in the sense of the digital redesign.

The following section presents a simple solution for stability preservation without affecting the zeros and irrespective of the direct term.

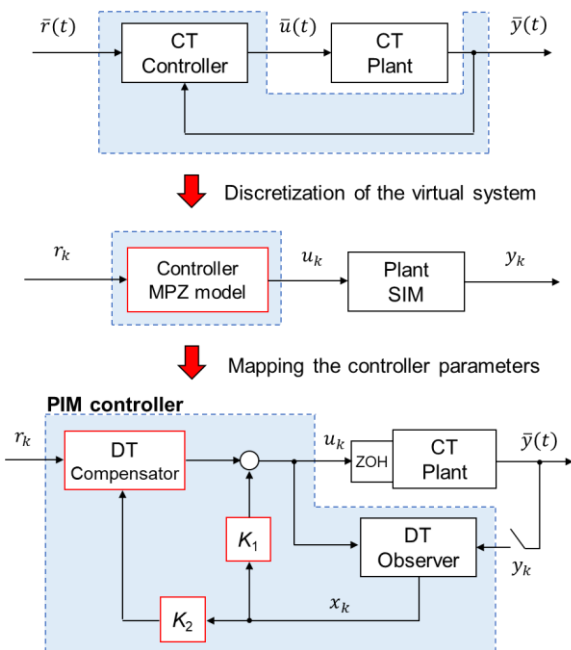


Fig. 1 Overview of the PIM discretization.

3. PROPOSED METHOD

3.1 Approach for the proposed discretization process

The conventional PIM method uses the MPZ model to discretize the PISE of the underlying system. The present study considers to employ the mapping zero model. The controller parameters are predetermined from the mapping zero model and then modified to be the desired ones through internal state feedback.

The essential difference between the conventional and proposed process is what is subject to the parameter modification. As explained in section 2.2, the discretization of the MPZ model includes the pole-placement modification. Thus, The conventional PIM method modifies the reference model in advance of the parameter matching. On the other hand, in the proposed process, the prescribed digital feedback controller is modified after the parameter matching. This order is reasonable because while the parameter modification in the MPZ discretization is achieved in a single block manner, the proposed process can take the feedback structure into account. The following subsection explains the discretization algorithm with reference to Fig.2.

3.2 Proposed discretization algorithm

The proposed algorithm employs the mapping zero model to discretize the CT-PISE described by eqn. (9), which is given by

$$\delta \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} = \begin{bmatrix} A_{MZ,11} & A_{MZ,12} \\ A_{mz,21} & A_{mz,22} \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + \begin{bmatrix} B_{MZ,1} \\ B_{MZ,2} \end{bmatrix} \Gamma r_k \quad (14)$$

$$u_k = \begin{bmatrix} C_{MZ,1} & C_{MZ,2} \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + D_{MZ} \Gamma r_k$$

The parameters of the PIM digital controller, given by eqn. (11), are determined by matching with eqn. (14) as

$$A'_C = A_{MZ,22}, B'_C = B_{MZ,2}, C'_C = C_{MZ,2}, D'_C = D_{MZ}, \quad (15)$$

$$K'_1 = -C_{MZ,1}, K'_2 = -A_{MZ,21}.$$

The determined parameters compose the DT PISE, which is described by

$$\delta \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} = \begin{bmatrix} A_p - B_p K'_1 & B_p C'_C \\ -K'_2 & A'_C \end{bmatrix} \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + \begin{bmatrix} B_p D'_C \\ B'_C \end{bmatrix} \Gamma r_k \quad (16)$$

$$u_k = [-K'_1 \quad C'_C] \begin{bmatrix} x_{P,k} \\ x_{C,k} \end{bmatrix} + D'_C \Gamma r_k$$

This DT system is then modified to have the desired pole placements. Let the parameter matrices of eqn. (16) be denoted by $(A'_{PISE}, B'_{PISE}, C'_{PISE}, D'_{PISE})$. When the CT feedback system is controllable, the state equation (A'_{PISE}, B'_{PISE}) is also controllable. Thus, there exists a state feedback gain which places the poles of the DT system arbitrary. The target placement is designated by the step-invariant model of the CT PISE. Let the state feedback gain achieving the placement for the DT system be denoted by $K_f = [K_{f,1} \quad K_{f,2}]$. Then, the parameter matrices are modified by

$$\begin{aligned}
A_{PISE} &= \begin{bmatrix} A_p - B_p(K'_1 + D'_c K_{f,1}) & B_p(C'_c - D'_c K_{f,2}) \\ -K'_2 - B'_c K_{f,1} & A'_c - B'_c K_{f,2} \end{bmatrix} \\
B_{PISE} &= B'_{PISE} \\
C_{PISE} &= [-K'_1 - D'_c K_{f,1} \quad C'_c - D'_c K_{f,2}] \\
D_{PISE} &= D'_{PISE}
\end{aligned} \quad (17)$$

Thus, the PIM digital controller is finally determined by

$$\begin{aligned}
A_C &= A'_c - B'_c K_{f,2}, \quad B_C = B'_c, \\
C_C &= C'_c - D'_c K_{f,2}, \quad D_C = D'_c, \\
K_1 &= K'_1 + D'_c K_{f,1}, \quad K_2 = K'_2 + B'_c K_{f,1}.
\end{aligned} \quad (18)$$

3.3 On the guaranteed stability

The guaranteed stability of the PIM discretization are based on the following two propositions:

- (i) The PISE of the digital feedback system has the zeros (transmission zeros) which exactly correspond to the poles of the step invariant model of the plant.
- (ii) The PISE of the digital feedback system has the poles which are mapped into the DT domain in accordance with the same rule as the step invariant model.

Since the proposition (ii) can hold simply when the PISE of the digital controller is controllable, we explain that the proposed method satisfies the proposition (i) even though the poles are modified after parameter matching.

Proposition (i) can be confirmed by the controllability of the extended system obtained by connecting the PISE to the plant model in series. Let us define the state vector of the plant connected to the PISE as $x'_{p,k}$ for distinction from the state vector of the plant included in the PISE. Then, the extended system can be written by

$$\delta \begin{bmatrix} x'_{p,k} \\ x_{p,k} \\ x_{c,k} \end{bmatrix} = \begin{bmatrix} A_{SIM} & -B_{SIM}K_1 & B_{SIM}C_C \\ 0 & A_{SIM} - B_{SIM}K_1 & B_{SIM}C_C \\ 0 & -K_2 & A_C \end{bmatrix} \begin{bmatrix} x'_{p,k} \\ x_{p,k} \\ x_{c,k} \end{bmatrix} + \begin{bmatrix} B_{SIM}D_C \\ B_{SIM}D_C \\ B_{SIM} \end{bmatrix} r_k. \quad (19)$$

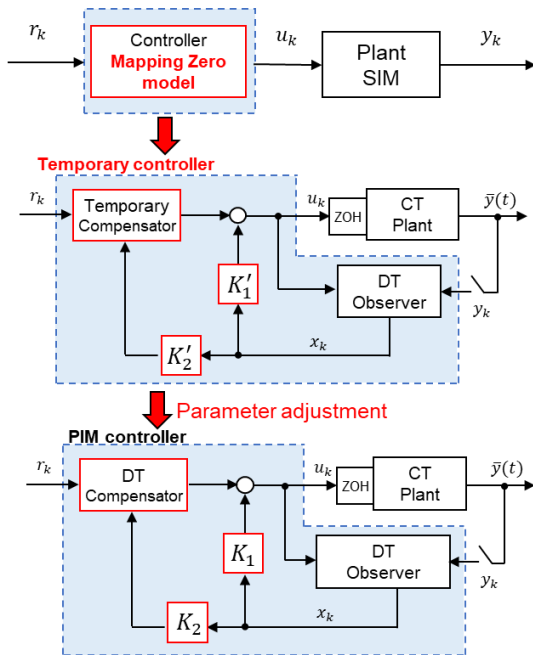


Fig. 2 The proposed discretization process.

The controllability of this extended system, notated by (A_{EXT}, B_{EXT}) , is confirmed by Popov-Belevitch-Hautus (PBH) test [16]. Let a matrix $Q(\varepsilon) = [\varepsilon I - A_{EXT} \quad B_{EXT}]$ be considered, where $\varepsilon \in \mathbb{C}$. Applying the elementary matrix operations,

$$Q \rightarrow \begin{bmatrix} \varepsilon I - A_{SIM} & 0 & 0 & 0 \\ 0 & \varepsilon I - (A_{SIM} - B_{SIM}K_1) & -B_{SIM}C_C & B_{SIM}D_C \\ 0 & -K_2 & \varepsilon I - A_C & B_C \end{bmatrix} \quad (20)$$

Obviously, the row rank of the matrix is reduced when the plant poles are substituted to ε . Therefore, the digital controller of the proposed PIM method includes the transmission zeros eliminating the plant poles, which holds irrespective of the state feedback gain K_f .

4. NUMERICAL EXAMPLE

4.1 Underlying system

The proposed discretization algorithm is first verified through an academic example. Let us consider the discretization of the closed loop system consisting of the plant described by

$$\begin{aligned}
\dot{\bar{x}}_p(t) &= \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \bar{x}_p(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bar{u}(t), \\
\bar{y}(t) &= [1 \quad -1] \bar{x}_p(t),
\end{aligned} \quad (21)$$

and the compensator given by

$$\begin{aligned}
\dot{\bar{x}}_c(t) &= \begin{bmatrix} 0 & 1 \\ -3 & -3 \end{bmatrix} \bar{x}_c(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\bar{r}(t) - \bar{y}(t)), \\
\bar{u}(t) &= [1 \quad 2] \bar{x}_c(t).
\end{aligned} \quad (22)$$

The plant is a non-minimum phase system, and the compensator has no direct term. The resulting feedback system is stable, which is described by

$$\begin{aligned}
\begin{bmatrix} \dot{\bar{x}}_p \\ \dot{\bar{x}}_c \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -5 & -4 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -3 & -3 \end{bmatrix} \begin{bmatrix} \bar{x}_p \\ \bar{x}_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \bar{r}, \\
\bar{u} &= [0 \quad 0 \quad 1 \quad 2] \begin{bmatrix} \bar{x}_p \\ \bar{x}_c \end{bmatrix}.
\end{aligned} \quad (23)$$

4.2 Discretization by the proposed method

When the sampling interval is selected as $T = 0.01$, the step-invariant model of the plant is obtained by

$$\begin{aligned}
\delta x_{p,k} &= \begin{bmatrix} -0.02 & 0.98 \\ -4.90 & -3.95 \end{bmatrix} x_{p,k} + \begin{bmatrix} 0.00 \\ 0.98 \end{bmatrix} u_k, \\
y_k &= [1 \quad -1],
\end{aligned} \quad (24)$$

and the mapping zero model of the PISE is obtained by

$$A_{MZ} = \begin{bmatrix} -0.02 & 0.98 & 0.01 & 0.01 \\ -4.91 & -3.94 & 0.97 & 1.97 \\ -0.00 & 0.00 & -0.00 & 0.99 \\ -1.00 & 1.00 & -3.00 & -3.00 \end{bmatrix}, \quad B_{MZ} = \begin{bmatrix} -0.00 \\ 0.01 \\ 0.00 \\ 1.00 \end{bmatrix}, \quad (25)$$

$$C_{MZ} = [0 \quad 0 \quad 1 \quad 2], \quad D_{MZ} = 0$$

and the parameters are temporarily mapped into the prescribed digital controller. The state feedback gain for parameter modification is determined by comparing the

poles of the DT PISE of the PIM controller with those of the step invariant model of the CT PISE, given by

$$K_f = [0.04 \quad 0.06 \quad -0.04 \quad -0.03]. \quad (26)$$

Applying the modification, the proposed algorithm finally determines the digital controller as

$$A_c = \begin{bmatrix} -0.00 & 0.99 \\ -2.96 & -2.96 \end{bmatrix}, B_c = \begin{bmatrix} 0.00 \\ 1.00 \end{bmatrix}, C_c = [1 \quad 2], D_c = 0, \quad (27)$$

$$K_1 = [0.00 \quad 0.00], K_2 = \begin{bmatrix} 0.00 & -0.00 \\ 1.04 & -0.94 \end{bmatrix}.$$

The constant gain is determined by the dc gain of the underlying CT feedback loop and the discretized feedback loop, which is calculated as $\Gamma = 0.988$.

Since an arbitrary observer can be employed, we adopt a Kalman filter for state estimation to simplify the observer design which may impose trial-and-error gain tuning on the designer. The covariances of the process noise Q and observation noise R are set to $0.001I$ and 0.001 , respectively. The initial state estimate $x_{P,0}$ and estimation error covariance P_0 are set to is set to $\mathbf{0}$ and the identity matrix, respectively.

4.3 Result

The unit step response of the obtained controller is examined through a simulation as shown in Fig. 3, where the response is compared with that of the underlying feedback controller and the conventional PIM digital controller with an additional non-singular direct term ($\bar{D}_c = 2$ is added before the discretization). The poles and zeros of the proposed PIM digital controller are placed in accordance with the certain mapping rule. The emulation performance in the transient response is obviously improved.

5. EXPERIMENT

5.1 Experimental setup

The proposed PIM method is then evaluated through an experiment using a commercial stepping motor (PK566H-B: Orientalmotor Inc.) and driver (ADB5410: Melec Corporation). The current regulation module of the motor drive is a target for the discretization. Since the current regulator is an electronic system, it requires a smaller sampling interval for stable digital implementation than a mechanical system. A digital signal processor (sBOXII: MIS corporation) is used to execute the control algorithm and capture the driver signals. The rotation speed and angle can be recorded by an encoder (M-1S: Canon Inc.).

The block diagram of the current regulator is shown in Fig. 5. The current regulator employs an integrator with high feedback gain. The plant consists of a part of drive circuit (PWM signal generator, driving capacitor) and motor coils, which has been modeled by a third order system described by

$$\dot{\bar{x}}_p = \begin{bmatrix} -4.50 \times 10^3 & -8.20 \times 10^6 & -4.30 \times 10^9 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \bar{x}_p + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \bar{u} \quad (28)$$

$$\bar{y} = [2.52 \times 10^4 \quad 8.75 \times 10^6 \quad 1.75 \times 10^{10}] \bar{x}_p$$

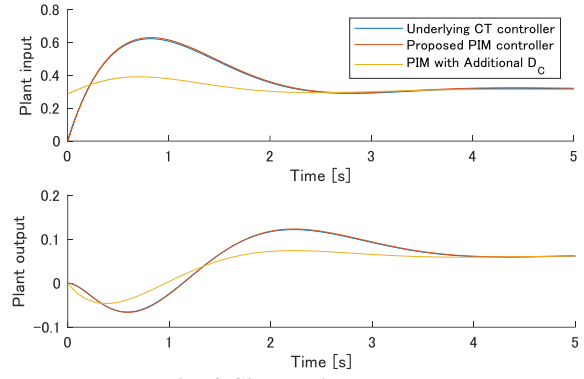


Fig. 3 Simulation result.

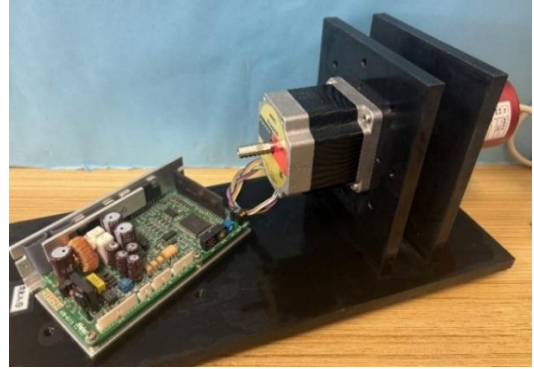


Fig. 4 Stepping motor and analog driver.

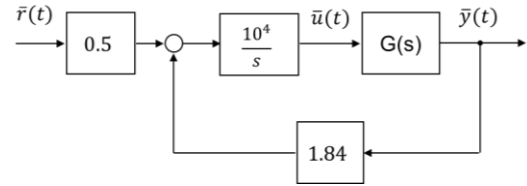


Fig. 5 Block diagram of the underlying regulator.

Thus, the CT PISE is given by

$$\begin{bmatrix} \dot{\bar{x}}_p \\ \dot{\bar{x}}_c \end{bmatrix} = \begin{bmatrix} -4.5 \times 10^3 & -8.2 \times 10^6 & -4.3 \times 10^9 & 10^4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -4.6 \times 10^4 & -1.6 \times 10^7 & -3.2 \times 10^{10} & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_p \\ \bar{x}_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix} \bar{r} \quad (29)$$

$$\bar{u} = [0 \quad 0 \quad 0 \quad 10^4] \begin{bmatrix} \bar{x}_p \\ \bar{x}_c \end{bmatrix}$$

5.2 Discretization

To illustrate the advantage of the proposed PIM discretization algorithm, the sampling interval was set to $200 \mu\text{s}$. This interval corresponded to almost twice of the system bandwidth, and we confirmed that a representative conventional technique, Tustin's method, could not achieve stable digital implementation of the current regulator under this condition.

The proposed PIM discretization algorithm gives the digital form of the underlying system as

$$A_c = -1.10 \times 10^{-4}, B_c = 0.5, C_c = 10^4, D_c = 0, \quad (30)$$

$$K_1 = [0 \quad 0 \quad 0], K_2 = [3.09 \times 10^3 \quad -5.39 \times 10^6 \quad 1.33 \times 10^9],$$

The constant gain is determined by comparison of the dc gain of each feedback loop, calculated as $\Gamma = 1.5$. The Kalman filter is employed for state estimation as well as the previous example, whose design parameters are determined as $Q = 0.001 I$, $R = 0.001$, $x_{P,0} = \mathbf{0}$, and $P_0 = I$.

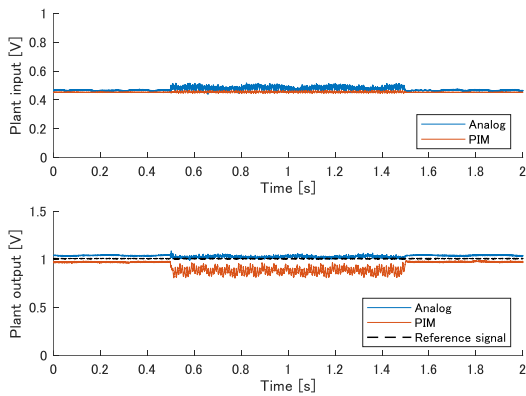


Fig. 6 Experimental results: the driver signals.

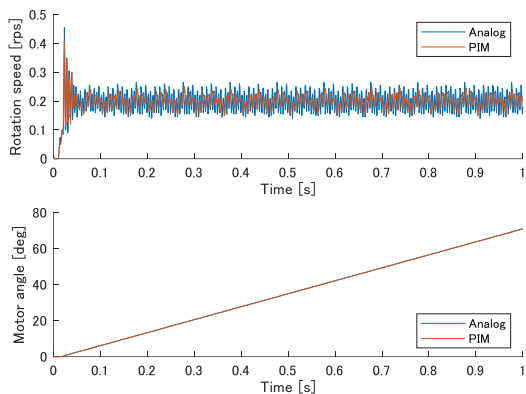


Fig. 7 Experimental results: the rotation data.

5.3 Experimental result

As the experimental condition, the pulse rate and number are set to 100 Hz and 100 respectively. In the experimental setup, one pulse corresponds to 0.72 deg, and therefore, the driving condition corresponds to the rotation from 0 to 72 deg in one second.

The experimental results are shown in Figs. 6 and 7. The driver signals, the plant input and output, were captured for 2 sec, which include the hold and rotation modes. The motor starts rotating from 0.5 sec in this figure, and the speed and angle are as shown in Fig. 7.

As indicated by these figures, the PIM digital controller is successfully implemented even with a large sampling interval, and the stepping motor rotates step by step without occurrence of loss-of-synchronism. On the other hand, as indicated by non-zero system matrix in eqn. (30), the discretized controller loses the integration property, which yields the regulation error in the plant output signal during rotation. This is a representative problem of global discretization; while the closed-loop poles are dealt with in an appropriate manner, each single block property cannot be ensured. Establishing an appropriate prescribed form and parameter modification algorithm for a servo system remains in future works.

6. CONCLUSION

This paper proposes a new plant-input-mapping discretization algorithm applicable to the system lacking a direct term of the compensator. Without respect to the

direct term, the proposed algorithm can derive a digital form of the underlying feedback system with preserving its stability for any non-pathological sampling interval. The proposed method is verified through an academic example, and the stable digital implementation even with a comparatively low sampling rate is demonstrated using a commercial motor current regulator. Although the problem of the preservation of the integrator property remains in future works, the extension of the PIM method to a strictly proper feedback system is achieved.

ACKNOWLEDGMENT

The experiment conducted in this study has received financial and technical support from Melec Corporation.

REFERENCE

- [1] Y. Yamamoto, "New approach to sampled-data control systems—a function space method," in *29th IEEE Conference on Decision and Control*, Honolulu: IEEE, 1990, pp. 1882–1887.
- [2] R. Middleton and G. Goodwin, "Improved finite word length characteristics in digital control using delta operators," *IEEE Trans. Automat. Contr.*, vol. 31, no. 11, pp. 1015–1021, 1986.
- [3] R. F. Whitbeck and L. G. Hofmannf, "Digital Control Law Synthesis in the w Domain," *J. Guid. Control. Dyn.*, vol. 1, no. 5, pp. 319–326, Sep. 1978.
- [4] A. H. D. Markazi and N. Hori, "A New Method with Guaranteed Stability for Discretization of Continuous-Time Control Systems," *Proc. Am. Control Conf. Chicago, June*, pp. 1397–1402, 1992.
- [5] M. K. Sain and C. B. Schrader, "The role of zeros in the performance of multiinput, multioutput feedback systems," *IEEE Trans. Educ.*, vol. 33, no. 3, pp. 244–257, 1990.
- [6] N. Hori, R. Cormier, and K. Kanai, "On matched pole-zero discrete-time models," *IEE Proc. D Control Theory Appl.*, vol. 139, no. 3, p. 273, 1992.
- [7] N. Nishinaga, N. Hori, and X. Ruan, "Design of Dual-Rate PIM-Disturbance Regulators for a DC-DC Switching Converter," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 646, p. 012019, Oct. 2019.
- [8] I. Sejimo and N. Hori, "Models for PIM Digital Redesign of a Stepping Motor Current Regulator," *Proc. SICE Annu. Conf. 2004*, pp. 355–360, 2004.
- [9] A. H. D. Markazi, "Matched pole-zero state-space model and continuous-time properties," *Sci. Iran.*, vol. 20, no. 4, pp. 1193–1201, 2013.
- [10] K. Yagi, "Matched-pole-zero discrete-time model in the state-space representation," *IET Control Theory Appl.*, vol. 14, no. 19, pp. 3270–3281, Dec. 2020.
- [11] S. Kawai and K. Yagi, "Matched Pole–Zero Model Independent of the Properness of Systems," *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 1722–1727, 2023.
- [12] K. Yagi, H. Muto, and Y. Mori, "Plant-Input-Mapping Discretization Method for a Feedback System in the State-Space Form," *J. Dyn. Syst. Meas. Control*, vol. 143, no. 12, p. 121004 (9pages), Dec. 2021.
- [13] S. Sawahata and K. Yagi, "Model Following Approach for Plant-Input-Mapping Discretization of a Current Regulator for Stepping Motors," *2024 SICE Festiv. with Annu. Conf. SICE FES 2024*, pp. 895–900, 2024.
- [14] T. Mori, P. N. Nikiforuk, M. M. Gupta, and N. Hori, "A class of discrete-time models for a continuous-time system," *IEE Proc. D Control Theory Appl.*, vol. 136, no. 2, pp. 79–83, 1989.
- [15] R. Middleton and J. Freudenberg, "Non-pathological sampling for generalized sampled-data hold functions," *Automatica*, vol. 31, no. 2, pp. 315–319, 1995.
- [16] T. Kailath, *Linear Systems*. Prentice Hall, 1980.