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Wireless Node Localization Using Likelihood Function Based on Distance Ratio Derived from RSSI

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Abstract: In recent years, with the rise of information-based societies, smart cities have been proposed, and Wireless Sensor Network (WSN) have attracted attention as a means of collecting information. In addition, the location information of each node is essential for the operation of WSN, so it is necessary to consider the localization system to be installed. A localization system for a WSN must be usable both indoors and outdoors, and it should be compact and power-efficient. In this study, we aimed to improve the localization accuracy over a wide range by applying the Apollonius circular trajectory theory to RSSI (Received Signal Strength Indicator) from three or more localization anchor nodes and performing maximum likelihood estimation for the likelihood function. As a result, in simulation verification, the localization error was reduced by approximately 79.7% compared to conventional methods. However, increased distance between the nodes leads to a larger discrepancy between the true and estimated RSSI distance ratio values, which in turn degrades localization accuracy. In addition, since multipath fading and antenna directivity characteristics are not taken into account, further consideration, including experiments, is required.

Keywords: Wireless Sensor Network (WSN), Received Signal Strength Indicator (RSSI), localization, Apollonius circle

1. INTRODUCTION

Recently, the concept of an ultra-smart society, exemplified by Society 5.0 and smart cities, has been proposed[1]. WSN (Wireless Sensor Network) have gained attention as a means of collecting environmental information. A WSN consists of multiple wireless nodes equipped with sensors, which are placed in various locations and form a network through ad hoc communication to collect environmental and physical information. The location information of each node is essential for this system, and research is being conducted on localization technique that is small, low-power, and can be used indoors[2].Applications of this localization technique are expected to include obtaining data on people flow indoors, navigation services, and infrastructure and environmental monitoring.

Currently, satellite positioning technique such as GPS are widely used for outdoor positioning. However, satellite positioning technique are not suitable for localization in WSN because of their high power consumption and their inability to be used in environments where satellite signals cannot be received due to obstructions.

We have proposed a localization method based on the RSSI distance ratio obtained from the RSSI (Received Signal Strength Indicator) value when wireless signals emitted by the target node for localization (localization node), are received by three or more anchor nodes, whose installation locations are known. Here, RSSI is the signal strength from the localization node received by the anchor node, and it attenuates as the communication distance increases. In addition, the RSSI distance is the distance between the localization node and the anchor node derived from the Friis propagation formula, and the RSSI distance ratio is the ratio of the RSSI distances of the two different anchor nodes.

Previous research methods used four or more anchor nodes to achieve high-precision localization within an area of approximately 3m square in simulations[3]. However, for the localization system to be put into practical use, it is necessary to further expand the high-precision localization area. In conventional method, the RSSI distance ratio was calculated after acquiring the RSSI, and the intersections of the Apollonius circle locus were used as candidate points for the localization node. However, the more the localization area is expanded, the more the Apollonius circle locus fluctuates due to the influence of noise, so the distribution of intersections becomes sparser at the distance of the anchor node, or it becomes impossible to obtain intersections at all.

Therefore, the purpose of this report is to create a new likelihood function that can achieve high-precision localization even when the node is far from the anchor node, and to further expand the high-precision localization area.

2. BASIC PRINCIPLES OF LOCALIZATION

This chapter explains the basic principles of positioning based on the RSSI distance ratio. When a localization node and an anchor node are present in space, the relationship between RSSI (P_r) and communication distance is expressed by the Friis propagation formula shown in Eq.(1).

$$P_r = \frac{k}{d^2}, \quad k = G_t G_r P_t \left(\frac{\lambda}{4\pi}\right)^2 \tag{1}$$

The Friis propagation formula includes the communication distance d and wireless node-specific parameters rep-

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resented by k (transmission gain G_t, reception gain G_r, transmission power P_t, and wavelength λ). A method has been proposed to perform trilateration by using this formula to back-calculate the distance from the RSSI obtained at three different anchor nodes[4]. However, to calculate the distance from RSSI, it is necessary to know the node-specific parameters of the equipment to be used must be investigated in advance. To address this problem, we have proposed a method to eliminate k by using the RSSI distance ratio.

Consider the distance ratio when two anchor nodes with the same G_r and a localization node are lined up in a straight line. As shown in Fig.1, when a localization node is between anchor nodes A and B, the distance from anchor node A to the localization node is d_A, and the distance from anchor node B to the localization node is d_B. In this case, the distance ratio is $d_A : d_B$, and the normalized distance ratio is expressed as Eq.(2).

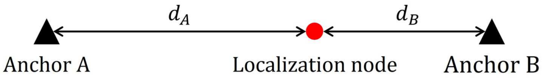

Fig. 1 1-dimensional distance ratio.

$$\mu = \frac{d_A}{d_A + d_B} \tag{2}$$

Here, if d_A and d_B are expressed using Eq.(1), it becomes as shown in Eq.(3). This makes it possible to eliminate the node-specific k value, and the RSSI distance ratio μ to the localization node can be calculated from the RSSI values of anchor nodes A and B.

$$\mu = \frac{\sqrt{\frac{k}{P_{rA}}}}{\sqrt{\frac{k}{P_{rA}}} + \sqrt{\frac{k}{P_{rB}}}} = \frac{\sqrt{P_{r_B}}}{\sqrt{P_{r_A}} + \sqrt{P_{r_B}}} \tag{3}$$

3. CONSTRUCTION OF LIKELIHOOD FUNCTION

3.1. 1-dimensional likelihood function

Next, we consider the RSSI distance ratio in a real environment using a likelihood function. In a real environment, Gaussian noise is superimposed on the RSSI, and the likelihood of the true value of the RSSI distance ratio μ is expressed by some likelihood function as shown in Fig.2. Approximating this with a Gaussian function gives Eq.(4).

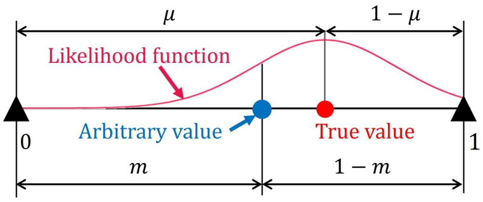

Fig. 2 1-dimensional likelihood function.

$$L_1(m) = \frac{\exp\left\{-\frac{(m-\mu)^2}{2\sigma^2}\right\}}{\int_0^1 \exp\left\{-\frac{(m-\mu)^2}{2\sigma^2}\right\} dm} \tag{4}$$

Here, m is an arbitrary RSSI distance ratio, μ is the true value of the RSSI distance ratio, and σ is the standard deviation of the RSSI distance ratio.

3.2. 2-dimensional likelihood function

We extend the one-dimensional likelihood function described in the previous section to a two-dimensional plane. In a two-dimensional plane, points with the same RSSI distance ratio exist on the Apollonius circle locus shown in Fig.3. Therefore, the likelihoods of points on the Apollonius circle locus are equal.

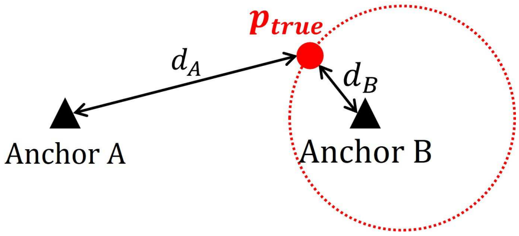

Fig. 3 Apollonius Circle.

Next, consider the likelihood at an arbitrary point $\boldsymbol{p}$ in a two-dimensional plane. As shown in Fig.4, there exists an Apollonius circular locus that passes through $\boldsymbol{p}$, and the likelihood at the intersection point $\boldsymbol{p\prime}$ with the line between the anchor nodes is equal to the likelihood at $\boldsymbol{p}$.

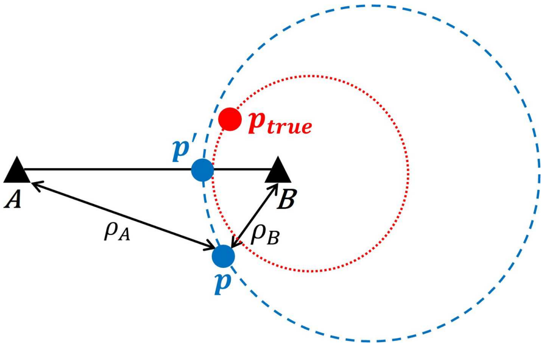

Fig. 4 Apollonius circular locus passing through arbitrary point $\boldsymbol{p}$.

From this, the likelihood $L_2(\boldsymbol{p})$ at $\boldsymbol{p}$ can be obtained from the likelihood function for the straight line between the anchor nodes, that is, Eq.(4), and is expressed as Eq.(5). Here, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the coordinates of anchor nodes A and B, and α is a damping term that prevents the coordinates with the maximum likelihood value from moving too far away from the anchor node; the smaller the value, the stronger the damping force. Theoretically, α is infinite, but taking into account the effects of RSSI observation noise, α takes a finite value.

$$\begin{aligned}
\rho_A(\boldsymbol{p}) &= \|\boldsymbol{p} - \boldsymbol{A}\|_2 \\
\rho_B(\boldsymbol{p}) &= \|\boldsymbol{p} - \boldsymbol{B}\|_2 \\
L_2(\boldsymbol{p}) &= \frac{1+\alpha}{\{\rho_A(\boldsymbol{p}) + \rho_B(\boldsymbol{p})\} + \alpha} \cdot \\
&\quad \frac{\sqrt{2}\exp\left[-\frac{1}{2\sigma^2}\left\{\frac{\rho_A(\boldsymbol{p})}{\rho_A(\boldsymbol{p})+\rho_B(\boldsymbol{p})} - \mu\right\}^2\right]}{\sqrt{\pi}\sigma\left\{\mathrm{erf}\left(\frac{\mu}{\sqrt{2}\sigma}\right) + \mathrm{erf}\left(\frac{1-\mu}{\sqrt{2}\sigma}\right)\right\}}
\end{aligned} \tag{5}$$

3.3. Localization by multiplying likelihood functions

In the previous section, we showed the likelihood function from two anchor nodes on a two-dimensional plane. However, with the likelihood function from two anchor nodes, the likelihood is constant on the Apollonius circle locus, and the localization coordinates cannot be uniquely determined (when α is infinity). Therefore, to perform localization, it is necessary to combine likelihood functions from three or more anchor nodes.

When N anchor nodes are placed, ${}_NC_2$ Apollonius circles will result. In Eq.(6), by multiplying the likelihood distributions that result for each pair of anchor nodes, we can obtain the likelihood function $L_p(\boldsymbol{p})$ for all combinations of anchor nodes. Using this likelihood function $L_p(\boldsymbol{p})$, the estimated coordinates can be calculated by performing maximum likelihood estimation.

$$\begin{aligned}
\rho_{Ai}(\boldsymbol{p}) &= \|\boldsymbol{p} - \boldsymbol{A_i}\|_2 \\
\rho_{Bi}(\boldsymbol{p}) &= \|\boldsymbol{p} - \boldsymbol{B_i}\|_2 \\
L_p(\boldsymbol{p}) &= \prod_{i=1}^{{}_NC_2}\left(\frac{1+\alpha}{\{\rho_{Ai}(\boldsymbol{p}) + \rho_{Bi}(\boldsymbol{p})\} + \alpha} \cdot \right. \\
&\quad \left. \frac{\sqrt{2}\exp\left[-\frac{1}{2\sigma^2}\left\{\frac{\rho_{Ai}(\boldsymbol{p})}{\rho_{Ai}(\boldsymbol{p})+\rho_{Bi}(\boldsymbol{p})} - \mu_i\right\}^2\right]}{\sqrt{\pi}\sigma\left\{\mathrm{erf}\left(\frac{\mu_i}{\sqrt{2}\sigma}\right) + \mathrm{erf}\left(\frac{1-\mu_i}{\sqrt{2}\sigma}\right)\right\}}\right)
\end{aligned} \tag{6}$$

4. SIMULATION VERIFICATION

Using simulated RSSI data, we verified the effectiveness of expanding the high-precision localization area with the likelihood function. The parameters used in the RSSI simulation are shown in Table 1. For the simulation, Gaussian noise (standard deviation P_σ) was added to the ideal radio wave model shown in Eq.(1). Also, since it is not possible to obtain μ_i in Eq.(6), positioning is performed by using the estimated true value $\hat{\mu}_i$. The estimated true value $\hat{\mu}_i$ used in the localization was calculated by averaging 50 RSSI distance ratio data, and the attenuation parameter α was set to 10000.

Table 1 Simulation parameters.

G_r [dBi]	G_t [dBi]	P_t [dBm]	λ [GHz]	P_σ [dBm]
1.64	2.00	2.50	2.4	−47.4

The results of a localization simulation performed within a 6m square area are shown in Fig.5. The results of applying the same RSSI to the conventional method[3] are shown in Fig.6.

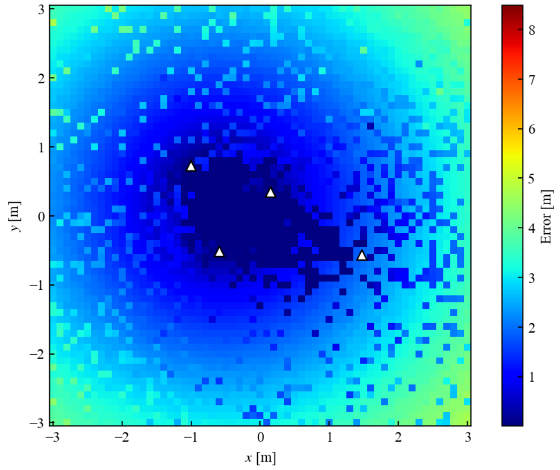

Fig. 5 Localization error distribution in the proposed method (four randomly placed anchor nodes).

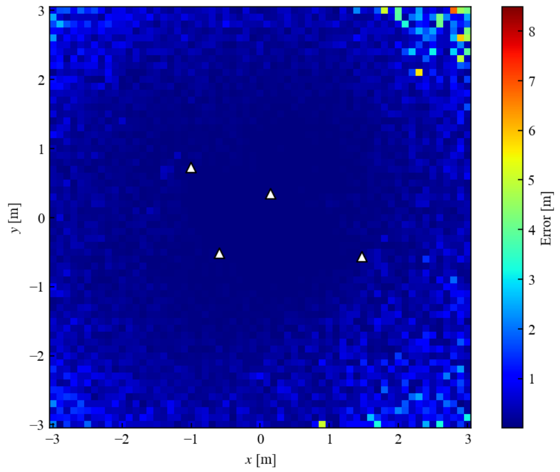

Fig. 6 Localization error distribution in the conventional method (four randomly placed anchor nodes).

In Fig.5 and Fig.6, a anchor nodes represented by a triangle are placed on the x-y plane, and the localization error is represented by color when the localization node is placed at each point of a grid with 0.1 m intervals. These results show that the localization method using the likelihood function is able to expand the high-precision localization area compared to conventional method.

Table 2 shows the results of a localization simulation when 4 to 6 anchor nodes were randomly installed in 10 allocation patterns. These results demonstrate that the proposed method significantly expands the high-precision localization area compared to the conventional method. Moreover, the use of a likelihood function based on the RSSI distance ratio enables high-precision localization over a broader region.

Table 2 Comparison of average errors between the proposed method and the conventional method.

Number of nodes	Conventional method [m]	Proposed method [m]	Improvement rate [%]
4	1.958	0.598	69.5
5	1.970	0.344	82.5
6	1.966	0.255	87.0

5. CONCLUSIONS

In this study, we constructed a likelihood function that uses the true value of the RSSI distance ratio to achieve high-precision localization over a wide area. As a result, in simulation verification, we were able to reduce the localization error by approximately 79.7% on average compared to conventional method. However, it was found that when localization at even wider range, the estimated true value of the RSSI distance ratio becomes inaccurate, resulting in a decrease in localization accuracy. This may be due to the fact that the RSSI distance ratio has a distorted distribution, making it impossible to accurately estimate the maximum likelihood RSSI distance ratio using averaging processing, and that the maximum likelihood RSSI distance ratio deviates from the true value as the communication distance increases.

To solve these problems, it is necessary to estimate the true value of the RSSI distance ratio by a method other than averaging. In addition, in this paper, localization was performed by maximum likelihood estimation using a likelihood function, but Bayesian estimation may be more effective in terms of noise resistance performance.

In the future, we will conduct experiments using an actual device to confirm the effects of multipath fading due to obstacles, and will also investigate a method of high-precision localization over a wider area by applying Bayesian estimation.

REFERENCES

[1] Cabinet Office Society 5.0 Website, https://www8.cao.go.jp/cstp/society5_0/, Referenced April 21, 2024.

[2] F. Zafari, A. Gkelias, K. K. Leung, “A Survey of Indoor Localization Systems and Technologies”, *IEEE Communications Surveys & Tutorials (Volume: 21, Issue: 3, thirdquarter 2019)*, pp. 2568–2599, 2019.

[3] H. Shimmura, J. Takayama, “Expansion of Localization Area Using Intersection Group Selection in Wireless Node Localization Based on RSSI Ratio”, *SICE Festival 2024 with Annual Conference*, pp. 862–865, 2024.

[4] R. Mardeni, O. S. Nizam, “Node Positioning in ZigBee Network Using Trilateration Method Based on the Received Signal Strength Indicator (RSSI)”, *European Journal of Scientific Research Vol.46 No.1*, pp. 48–61, 2010.