Proceedings of the 2025 SICE Festival with Annual Conference
September 9-12, 2025, Chiang Mai, Thailand

Stabilization of Linear Input-Delay Systems based on Data Informativity

Kohei Ayaka[1†], Takumi Namba[1], and Kiyotsugu Takaba[1]

[1]Department of Electrical and Electronic Engineering, Ritsumeikan University, Shiga, Japan
(E-mail: {re0164si@ed., t-namba@fc., ktakaba@fc.}ritsumei.ac.jp)

Abstract: This paper proposes a novel data-driven method for stabilization of a linear system with input delay based on the notion of data informativity. We derive a linear matrix inequality (LMI) condition for the data informativity for quadratic stabilization by considering the structure of the augmented state equation of the input-delay system. The LMI condition enables us to efficiently design the augmented state-feedback controller, which stabilizes every linear input-delay system conforming to a given data. The effectiveness of the proposed method is verified through a numerical simulation.

Keywords: input delay, data-driven control, data informativity, quadratic stabilization, linear matrix inequality

1. INTRODUCTION

In recent years, the development of AI and data science has been facilitating the use of increasing amounts of data. Data-driven control has been attracting great attention since it effectively designs a controller *directly* from plant response data.

Although model-based control enables us to capture the physical characteristics of the system under consideration, it is difficult to apply in the presence of uncertainties and can be time-consuming due to the system identification step. In contrast, data-driven control enables better handling of uncertainties from data and reduces the time required for system identification, while the physical characteristics may not be clearly captured.

Several studies have been reported on direct data-driven control [1-4]. In particular, van Waarde *et al.* [2, 3] proposed the notion of *data informativity* which characterizes the necessary and sufficient amount of data for achieving a given control objective, such as stabilization. They also extended the previous results to the case of noisy data and derived a condition for the data-driven synthesis of a stabilizing controller [4].

This paper is concerned with the data-driven design of a stabilizing controller for a linear input-delay system. Linear input-delay systems can be encountered in various applications such as remote control systems. If the delay length is known *a priori*, it is a common technique to represent the input-delay systems by an augmented state equation, in which the state vector is extended to include the delayed inputs (see e.g. [5, 6]). Even in the data-driven approach, this structure should be properly reflected in the control design to achieve a given control objective. Therefore, we will derive a linear matrix inequality (LMI) condition for the data-driven design of an *augmented* state-feedback stabilizing controller for the linear input-delay system based on the data informativity.

As a related work, R.-Escobedo, Fridman, and Schiffer [7] derived sufficient LMI conditions for data-driven design of a state-feedback stabilizing controller for a linear system with unknown time delays based on the robust control theory. The main difference between the present work and the reference [7] is that we design the augmented state-feedback controller, including the delayed input values, rather than the state-feedback controller. This is because we take into account the augmented state structure of the input-delay system with a known delay length. Nevertheless, our LMI condition will be *compact* in the sense that it is derived using the input-state data of the original input-delayed state equation, not those of the augmented state equation.

The organization of this paper is as follows. In Section 2, we will briefly review the description of a linear input-delay system in terms of input-state data and the notion of data informativity. In Section 3, exploiting the structure of the augmented state equation, we will formulate the data-driven stabilization of a linear input-delay system as a quadratic stabilization problem for a certain set of models conforming to the data. Then, we will derive a necessary and sufficient condition for informativity for the quadratic stabilization in terms of LMIs. We will verify the effectiveness of the proposed design through numerical simulations in Section 4. We will give concluding remarks in Section 5.

2. DATA-BASED SYSTEM DESCRIPTION

Consider a linear discrete-time system with input delay defined by the state equation

$$x(t+1) = A_s x(t) + B_s u(t-d) + w(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $w(t) \in \mathbb{R}^n$ are the state, control input, and noise at time t, respectively. d is the length of the input delay. The matrix pair $(A_s, B_s) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$ represents the nominal plant model that is not available to the designer.

Assumption 1. *The state dimension n and the delay length d are known a priori.*

One possible scenario for obtaining prior knowledge of the delay length is a situation where transient response data, such as a step response from the steady state, is available. In this case, the delay length can be identified

† Kohei Ayaka is the presenter of this paper.

from the time shift between the input and state trajectories.

We form the data matrices by collecting the state and input sequences over the interval $[t_0,\ T]$ as

$$
\begin{aligned}
U_- &= \begin{bmatrix} u(t_0-d) & u(t_0-d+1) & \cdots & u(t_0-d+T-1) \end{bmatrix} \in \mathbb{R}^{m\times T},\\
X_- &= \begin{bmatrix} x(t_0) & x(t_0+1) & \cdots & x(t_0+T-1) \end{bmatrix} \in \mathbb{R}^{n\times T},\\
X_+ &= \begin{bmatrix} x(t_0+1) & x(t_0+2) & \cdots & x(t_0+T) \end{bmatrix} \in \mathbb{R}^{n\times T},\\
W_- &= \begin{bmatrix} w(t_0) & w(t_0+1) & \cdots & w(t_0+T-1) \end{bmatrix} \in \mathbb{R}^{n\times T}.
\end{aligned}
$$

Then, any matrix pair (A, B) conformed to the data (X_+, X_-, U_-) must satisfy

$$X_+ = AX_- + BU_- + W_- \tag{2}$$

for some noise matrix W_-.

We make the following assumption on W_- to characterize the noise property [4].

Assumption 2. *For some given matrices* $\Phi_{11} = \Phi_{11}^\top$, Φ_{12}, $\Phi_{22} = \Phi_{22}^\top \prec 0$, *the matrix* W_- *satisfies*

$$\begin{bmatrix} I \\ W_-^\top \end{bmatrix}^\top \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \begin{bmatrix} I \\ W_-^\top \end{bmatrix} \succeq 0. \tag{3}$$

By combining (3) with (2), we get

$$\begin{bmatrix} I \\ A^\top \\ B^\top \end{bmatrix}^\top \begin{bmatrix} I & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix} \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \begin{bmatrix} I & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix}^\top \begin{bmatrix} I \\ A^\top \\ B^\top \end{bmatrix} \succeq 0. \tag{4}$$

Define the set of all models conformed to the data $\mathcal{D}$ by

$$\Sigma_{\mathcal{D}} := \{(A, B) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m} \mid (4) \text{ is satisfied.}\}$$

The notion of data informativity proposed by van Waarde *et al.* [2] provides a set-theoretic characterization of the response data necessary and sufficient to achieve a desired control objective such as stabilization. Let $\mathcal{S}$ be a desired control specification. Define $\Sigma_{\mathcal{S},K}$ as the set of systems that satisfy $\mathcal{S}$ by a given controller K. Then, the data informativity of a control system is stated in Definition 1.

Definition 1. *The data* $\mathcal{D}$ *is said to be informative for the control specification* $\mathcal{S}$ *if there exists a controller* K *such that* $\Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{S},K}$.

In particular, if we take "stabilization" as the specification $\mathcal{S}$, $\Sigma_{\mathcal{S},K}$ represents the set of all systems stabilized by K. The informativity in this case means that there exists a controller K which stabilizes every system in $\Sigma_{\mathcal{D}}$.

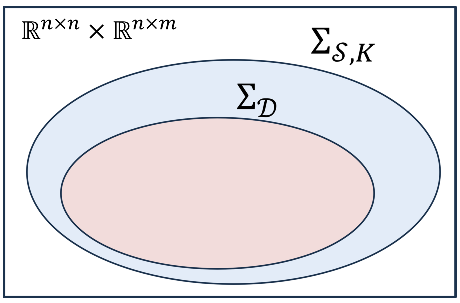


Fig. 1 Data informativity

3. DATA-DRIVEN STABILIZATION OF THE INPUT-DELAY SYSTEM

In this section, we will derive a *data-driven* stabilization condition for the input-delay system based on the notion of *data informativity* [2].

It is well known that, by augmenting the state vector, the input-delay linear system (1) can be expressed by the following equation [5, 6].

$$
\begin{bmatrix} x(t+1) \\ u(t-d+1) \\ \vdots \\ u(t-1) \\ u(t) \end{bmatrix}
=
\begin{bmatrix}
A & B & 0 & \cdots & 0 \\
0 & 0 & I_m & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & I_m \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix} x(t) \\ u(t-d) \\ \vdots \\ u(t-2) \\ u(t-1) \end{bmatrix}
+ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I_m \end{bmatrix} u(t) \tag{5}
$$

It is observed from this equation that the system matrix has a sparse structure, and that $\begin{bmatrix} A & B \end{bmatrix}$, which must be identified, appears only in the upper left block of the system matrix.

We apply the augmented state feedback controller

$$u(t) = K_0 x(t) + \sum_{i=1}^{d} K_i u(t-d+i-1), \tag{6}$$

to the augmented state equation (5) in order to stabilize the input-delay system, where K_i, $i = 0, 1, \dots, d$ is the gain to be designed. Then, the closed-loop system is expressed by

$$X(t+1) = \mathcal{A}X(t), \tag{7}$$

$$
X(t) = \begin{bmatrix} x(t) \\ u(t-d) \\ \vdots \\ u(t-2) \\ u(t-1) \end{bmatrix} \in \mathbb{R}^{n+md},\quad
\mathcal{A} = \begin{bmatrix}
A & B & 0 & \cdots & 0 \\
0 & 0 & I_m & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & I_m \\
K_0 & K_1 & K_2 & \cdots & K_d
\end{bmatrix}.
$$

Let us introduce the following Lyapunov inequality that guarantees the stability of the closed-loop system (7).

$$\mathcal{P} \succ \mathcal{A}\mathcal{P}\mathcal{A}^\top, \quad \mathcal{P} = \mathcal{P}^\top \succ 0 \tag{8}$$

In general, the Lyapunov matrix $\mathcal{P}$ depends on the model matrices $(A, B) \in \Sigma_{\mathcal{D}}$. Nevertheless, we will derive a numerically tractable condition for stabilization based on the quadratic stability, which claims a constant Lyapunov matrix common to all models $(A, B) \in \Sigma_{\mathcal{D}}$.

Define $\Sigma_{\mathrm{stab},\mathcal{P},K}$ as the set of all (A, B) for which the Lyapunov inequality (8) is satisfied for a positive definite matrix $\mathcal{P}$ and an augmented state feedback gain $K = [K_0\ K_1\ \cdots\ K_d] \in \mathbb{R}^{m\times(n+md)}$. In view of Definition 1, we define the data informativity for quadratic stabilization of the linear input-delay system as follows.

Definition 2. *The data $\mathcal{D} = (X_+, X_-, U_-)$ is said to be informative for quadratic stabilization if there exists a feedback gain $K = \begin{bmatrix} K_0 & K_1 & \cdots & K_d \end{bmatrix} \in \mathbb{R}^{m\times(n+md)}$ and a positive definite matrix $\mathcal{P} \in \mathbb{R}^{(n+md)\times(n+md)}$ satisfying $\Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathrm{stab},\mathcal{P},K}$, i.e., (8) is satisfied for all $(A,B) \in \Sigma_{\mathcal{D}}$.*

The data informativity for quadratic stabilization is equivalent to saying that all models in $\Sigma_{\mathcal{D}}$ are quadratically stabilizable by some feedback gain K.

Partition $\mathcal{P}$ as

$$\mathcal{P} = \left[\begin{array}{cc:cc} P_{00} & P_{10}^\top & \cdots & P_{d0}^\top \\ P_{10} & P_{11} & \cdots & P_{d1}^\top \\ \hdashline \vdots & \vdots & \ddots & \vdots \\ P_{d0} & P_{d1} & \cdots & P_{dd}^\top \end{array}\right] =: \left[\begin{array}{c:c} \mathcal{P}_a & \mathcal{P}_b^\top \\ \hdashline \mathcal{P}_b & \mathcal{P}_c \end{array}\right],$$

and define Z, L and $L_{k:l}$ by

$$Z = \begin{bmatrix} A & B \end{bmatrix}^\top,$$
$$L = \begin{bmatrix} L_0 & L_1 & \cdots & L_d \end{bmatrix} = \begin{bmatrix} K_0 & K_1 & \cdots & K_d \end{bmatrix}\mathcal{P},$$
$$L_{k:l} = \begin{bmatrix} L_k & L_{k+1} & \ldots & L_l \end{bmatrix}.$$

Then, $\mathcal{A}\mathcal{P}\mathcal{A}^\top$ is expressed by

$$\mathcal{A}\mathcal{P}\mathcal{A}^\top = \left[\begin{array}{c:cc} Z^\top \mathcal{P}_a Z & Z^\top \mathcal{P}_b^\top & Z^\top L_{0:1}^\top \\ \hdashline \mathcal{P}_b Z & \mathcal{P}_c & L_{2:d}^\top \\ L_{0:1} Z & L_{2:d} & L\mathcal{P}^{-1}L^\top \end{array}\right] =: \left[\begin{array}{c:c} Z^\top Q_a Z & Z^\top Q_b^\top \\ \hdashline Q_b Z & Q_c \end{array}\right]$$

Corresponding to the above partition, we further introduce a new partition $\mathcal{P}$ as $\mathcal{P} = \left[\begin{array}{c:c} \hat{\mathcal{P}}_a & \hat{\mathcal{P}}_b^\top \\ \hdashline \hat{\mathcal{P}}_b & \hat{\mathcal{P}}_c \end{array}\right]$, $\mathcal{P} - \mathcal{A}\mathcal{P}\mathcal{A}^\top \succ 0$ is given by

$$\left[\begin{array}{c:c} \hat{\mathcal{P}}_a & \hat{\mathcal{P}}_b^\top \\ \hdashline \hat{\mathcal{P}}_b & \hat{\mathcal{P}}_c \end{array}\right] - \left[\begin{array}{c:c} Z^\top Q_a Z & Z^\top Q_b^\top \\ \hdashline Q_b Z & Q_c \end{array}\right] \succ 0 \tag{9}$$

To stabilize all the models conformed to the data $\mathcal{D}$, we wish to derive a necessary and sufficient condition for (9) to hold for every $Z = [A\ B]^\top$ satisfying (4). To this aim, we devise a modified version of the matrix S-lemma [4] by expanding the block size of the matrices.

Lemma 1 (Expanded matrix S-lemma)**.** *Define*

$$\mathcal{S}_N := \left\{ Z \in \mathbb{R}^{m\times n} \;\middle|\; \begin{bmatrix} I_n \\ Z \end{bmatrix}^\top \begin{bmatrix} N_a & N_b^\top \\ N_b & N_c \end{bmatrix} \begin{bmatrix} I_n \\ Z \end{bmatrix} \geq 0 \right\} \subset \mathbb{R}^{m\times n}$$

for a symmetric matrix $N = \begin{bmatrix} N_a & N_b^\top \\ N_b & N_c \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}$ satisfying $N_c \leq 0$ and $\ker N_c \subseteq \ker N_b^\top$. Let also symmetric matrices P and Q be given as

$$P = \begin{bmatrix} P_a & P_b^\top \\ P_b & P_c \end{bmatrix} \in \mathbb{R}^{(n+\ell)\times(n+\ell)},$$
$$Q = \begin{bmatrix} Q_a & Q_b^\top \\ Q_b & Q_c \end{bmatrix} \in \mathbb{R}^{(m+\ell)\times(m+\ell)}, \quad Q_a \geq 0.$$

Then,

$$\begin{bmatrix} P_a & P_b^\top \\ P_b & P_c \end{bmatrix} \succ \begin{bmatrix} Z^\top Q_a Z & Z^\top Q_b^\top \\ Q_b Z & Q_c \end{bmatrix} \quad \forall Z \in \mathcal{S}_N \tag{10}$$

holds if and only if there exist constants $\alpha \geq 0$ and $\varepsilon > 0$ satisfying

$$\begin{bmatrix} P_a - \varepsilon I_n & 0 & P_b^\top \\ 0 & -Q_a & -Q_b^\top \\ P_b & -Q_b & P_c - Q_c \end{bmatrix} - \alpha \begin{bmatrix} N_a & N_b^\top & 0 \\ N_b & N_c & 0 \\ 0 & 0 & 0 \end{bmatrix} \geq 0. \tag{11}$$

Proof: Applying the well-known Schur complement formula, the inequality (10) holds true if and only if

$$P_c - Q_c \succ 0 \tag{12}$$

and

$$(P_a - Z^\top Q_a Z) - (P_b - Q_b Z)^\top (P_c - Q_c)^{-1}(P_b - Q_b Z) \succ 0.$$

The latter inequality is rewritten as

$$\begin{bmatrix} I_n \\ Z \end{bmatrix}^\top \begin{bmatrix} P_a - P_b^\top (P_c - Q_c)^{-1} P_b & P_b^\top (P_c - Q_c)^{-1} Q_b \\ Q_b^\top (P_c - Q_c)^{-1} P_b & -Q_a - Q_b^\top (P_c - Q_c)^{-1} Q_b \end{bmatrix} \begin{bmatrix} I_n \\ Z \end{bmatrix} \succ 0. \tag{13}$$

Since Q_a is positive semi-definite, (12) implies $-Q_a - Q_b^\top (P_c - Q_c)^{-1} Q_b \leq 0$. Then, from Theorem 13 in [4], the inequality (13) holds for any $Z \in \mathcal{S}_N$ if and only if there exist constants $\alpha \geq 0$ and $\varepsilon > 0$ satisfying

$$\begin{bmatrix} P_a - \varepsilon I_n & 0 \\ 0 & -Q_a \end{bmatrix} - \alpha \begin{bmatrix} N_a & N_b^\top \\ N_b & N_c \end{bmatrix} - \begin{bmatrix} P_b^\top \\ -Q_b^\top \end{bmatrix} (P_c - Q_c)^{-1} \begin{bmatrix} P_b & -Q_b \end{bmatrix} \geq 0$$

Applying the Schur complement formula again, we see that the above inequality together with (12) is equivalent to (11). This completes the proof. □

Let us define

$$\hat{\mathcal{P}}_b = \left[\begin{array}{c} \hat{\mathcal{P}}_{b0} \\ \hdashline \hat{\mathcal{P}}_{b1} \end{array}\right] := \left[\begin{array}{c} P_{10} \\ \vdots \\ P_{d-1,0} \\ \hdashline P_{d0} \end{array}\right],$$

$$\hat{\mathcal{P}}_c = \left[\begin{array}{c:c} \hat{\mathcal{P}}_{c00} & \hat{\mathcal{P}}_{c10}^\top \\ \hdashline \hat{\mathcal{P}}_{c10} & \hat{\mathcal{P}}_{c11}^\top \end{array}\right] := \left[\begin{array}{ccc:c} P_{11} & \cdots & P_{d-1,1}^\top & P_{d1}^\top \\ \vdots & \ddots & \vdots & \vdots \\ P_{d-1,1} & \cdots & P_{d-1,d-1} & P_{d,d-1}^\top \\ \hdashline P_{d1} & \cdots & P_{d,d-1} & P_{dd} \end{array}\right].$$

and the matrix Ψ by

$$\Psi = \begin{bmatrix} \Psi_a & \Psi_b^\top \\ \Psi_b & \Psi_c \end{bmatrix} := \left[\begin{array}{cc} I & X_+ \\ \hdashline 0 & -X_- \\ 0 & -U_- \end{array}\right] \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \left[\begin{array}{cc} I & X_+ \\ \hdashline 0 & -X_- \\ 0 & -U_- \end{array}\right]^\top \tag{14}$$

We then arrive at the main result of this paper from Lemma 1 and the Schur complement formula.

Theorem 1. *Under Assumptions 1 and 2, the data* $\mathcal{D} = (X_+, X_-, U_-)$ *is informative for quadratic stabilization if and only if there exist a symmetric matrix* $\mathcal{P} \in \mathbb{R}^{(n+md)\times(n+md)}$*, a matrix* $L \in \mathbb{R}^{m\times(n+md)}$*, and constants* $\alpha \geq 0, \varepsilon > 0$ *satisfying*

$$\begin{bmatrix} \hat{\mathcal{P}}_a-\varepsilon I_n-\alpha\Psi_a & -\alpha\Psi_b^\top & \hat{\mathcal{P}}_{b0}^\top & \hat{\mathcal{P}}_{b1}^\top & 0\\ -\alpha\Psi_b & -\mathcal{Q}_a-\alpha\Psi_c & -\mathcal{P}_b^\top & -L_{0:1}^\top & 0\\ \hat{\mathcal{P}}_{b0} & -\mathcal{P}_b & \hat{\mathcal{P}}_{c00}-\mathcal{P}_c & \hat{\mathcal{P}}_{c10}^\top-L_{2:d}^\top & 0\\ \hat{\mathcal{P}}_{b1} & -L_{0:1} & \hat{\mathcal{P}}_{c10}-L_{2:d} & \hat{\mathcal{P}}_{c11} & L\\ 0 & 0 & 0 & L^\top & \mathcal{P} \end{bmatrix} \succeq 0. \tag{15}$$

If this is the case, one of the feedback gains that quadratically stabilize the input-delay system for $(A, B) \in \Sigma_{\mathcal{D}}$ *is given by*

$$K = \begin{bmatrix} K_0 & K_1 & \cdots & K_d \end{bmatrix} = L\mathcal{P}^{-1}. \tag{16}$$

Sketch of Proof: By Lemma 1, the inequality (9) holds for every $(A, B) \in \Sigma_{\mathcal{D}}$ if and only if there exist $\alpha \geq 0$ and $\varepsilon > 0$ satisfying

$$\begin{bmatrix} \hat{\mathcal{P}}_a-\varepsilon I_n & 0 & \hat{\mathcal{P}}_b^\top\\ 0 & -\mathcal{Q}_a & -\mathcal{Q}_b^\top\\ \hat{\mathcal{P}}_b & -\mathcal{Q}_b & \hat{\mathcal{P}}_c-\mathcal{Q}_c \end{bmatrix} - \alpha\begin{bmatrix} \Psi_a & \Psi_b^\top & 0\\ \Psi_b & \Psi_c & 0\\ 0 & 0 & 0 \end{bmatrix}$$
$$= \left[\begin{array}{c|c|c|c} \hat{\mathcal{P}}_a-\varepsilon I_n-\alpha\Psi_a & -\alpha\Psi_b^\top & \hat{\mathcal{P}}_{b0}^\top & \hat{\mathcal{P}}_{b1}^\top\\ \hline -\alpha\Psi_b & -\mathcal{Q}_a-\alpha\Psi_c & -\mathcal{P}_b^\top & -L_{0:1}^\top\\ \hline \hat{\mathcal{P}}_{b0} & -\mathcal{P}_b & \hat{\mathcal{P}}_{c00}-\mathcal{P}_c & \hat{\mathcal{P}}_{c10}^\top-L_{2:d}^\top\\ \hat{\mathcal{P}}_{b1} & -L_{0:1} & \hat{\mathcal{P}}_{c10}-L_{2:d} & \hat{\mathcal{P}}_{c11}-L\mathcal{P}^{-1}L^\top \end{array}\right] \succeq 0.$$

We obtain the LMI (15) by applying the Schur complement formula to the above inequality, focusing on the bottom-right block on the left-hand side. This completes the proof. □

The advantage of Theorem 1 is that it provides a compact characterization of data informativity for quadratic stabilization in terms of the data $\mathcal{D}$ of the original input-delay state equation (1) rather than that of the augmented state equation (5).

Notice also that (15) is an LMI with respect to $\mathcal{P}$, L, α, and ε, since $\mathcal{P}_\bullet$, $\hat{\mathcal{P}}_\bullet$, and $\mathcal{Q}_\bullet$ ($\bullet = a, b, c$) consist of the block components of $\mathcal{P}$. Therefore, Theorem 1 enables us to efficiently design the stabilizing feedback gain K by an existing convex programming algorithm.

We conclude this section by summarizing the algorithm for the data-driven design of a quadratically stabilizing controller based on Theorem 1.

Algorithm 1 Controller Synthesis

Given: $n, m, d, \mathcal{D} = (X_+, X_-, U_-)$

1: Determine $\Phi_{11} = \Phi_{11}^\top$, Φ_{12}, and $\Phi_{22} = \Phi_{22}^\top \prec 0$ based on the assumed property of the noise w.

2: Compute the matrix Ψ by substituting $\mathcal{D}$, Φ_{11}, Φ_{12}, Φ_{22} into (14).

3: Solve the LMI (15) to obtain a feasible solution $(\mathcal{P}, L, \alpha, \varepsilon)$.

4: Obtain the desired feedback gain K by (16).

4. NUMERICAL EXAMPLE

Consider the linear input-delay system (1) with

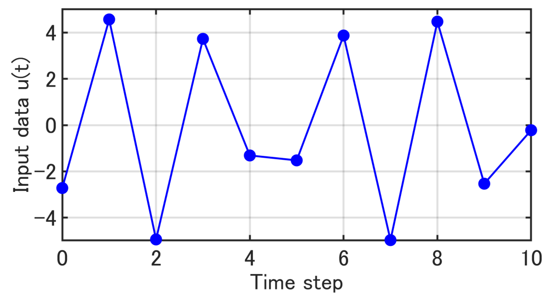
$$A_s = \begin{bmatrix} 1.3 & 0.5\\ 0 & 1.2 \end{bmatrix}, \quad B_s = \begin{bmatrix} 1\\ 1 \end{bmatrix}.$$

We assume that the delay length $d = 4$ has been identified *a priori* by some means.

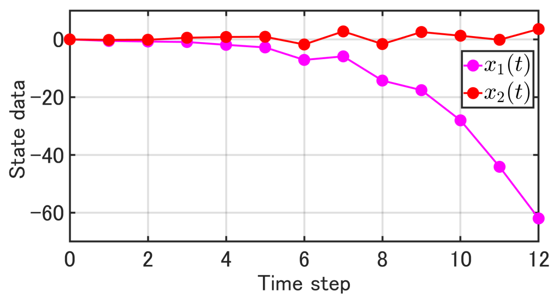
We collect the data $\mathcal{D} = (X_+, X_-, U_-)$ with $T = 10$ by injecting the sinusoidal input $u(t) = 5\sin 10t$ to the above model. The system is simulated under additive process noise, and the state evolves according to

$$x(t+1) = A_s x(t) + B_s u(t-d) + w(t),$$

where the process noise $w(t)$ is a zero-mean white Gaussian noise sequence which is independent and identically distributed with covariance $0.01 I_2$. The resulting noisy state trajectories along with the input signal are depicted in Fig. 2.



(a) Input data



(b) State data

Fig. 2 Input-state data $\mathcal{D}$

As for the inequality (3), we set $\Phi_{11} = \sigma^2 T I_2$, $\Phi_{12} = 0$, and $\Phi_{22} = -I_2$. This implies that the covariance of the noise $w(t)$ is not greater than $\sigma^2 I_2$. We need to carefully select the value of σ. If σ is too small, the model set $\Sigma_{\mathcal{D}}$ becomes empty or too small to contain the true system (A_s, B_s). Conversely, if σ is too large, $\Sigma_{\mathcal{D}}$ becomes so large that stabilizing all the models within it becomes very difficult. In this numerical example, the LMI (15) is feasible for $0.0990 \leq \sigma \leq 0.2608$. Note that we have executed Algorithm 1 using the SDP solver in MOSEK to obtain a feasible solution to (15).

We then choose $\sigma = 0.15$ to obtain the following feasible solution to the LMI (15).

$$\mathcal{P} = \begin{bmatrix} 0.3862 & 0.5324 & -0.5488 & -0.0722 & 0.0093 & 0.0175 \\ 0.5324 & 0.7980 & -0.6976 & -0.1395 & -0.0275 & -0.0051 \\ -0.5488 & -0.6976 & 1.0048 & -0.2333 & -0.0475 & -0.0116 \\ -0.0722 & -0.1395 & -0.2333 & 0.7152 & -0.2523 & -0.0561 \\ 0.0093 & -0.0275 & -0.0475 & -0.2523 & 0.6375 & -0.2291 \\ 0.0175 & -0.0051 & -0.0116 & -0.0561 & -0.2291 & 0.5866 \end{bmatrix}$$

$$L = [\,0.0117 \ \ -0.0029 \ \ 0.0001 \ \ -0.0153 \ \ -0.0472 \ \ -0.2011\,]$$

$$\alpha = 0.0095, \quad \varepsilon = 0.0002$$

Then, the desired stabilizing feedback gain K is given by

$$K = \begin{bmatrix} -20.8478 & 2.3194 & -11.6503 & -7.1128 & -4.1538 & -2.2330 \end{bmatrix}$$

It is verified that the closed-loop system $X(t+1) = \mathcal{A}X(t)$ with (A_s, B_s) is stabilized by this gain because the eigenvalues of $\mathcal{A}$ are

$$\{-0.7171, \ 0.8686, \ -0.3474 \pm 0.6615j, \ 0.4552 \pm 0.6748j\}$$

and all of them lie inside the unit disc as shown in Fig. 3. We also perform the simulation with the initial values

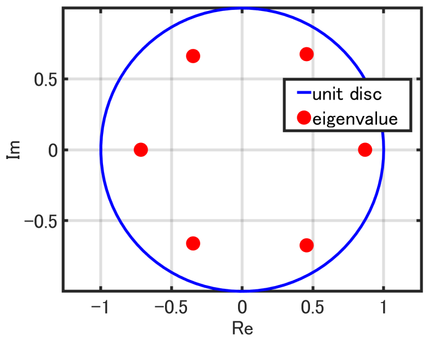


Fig. 3 Eigenvalues of closed-loop system for $A_s$and B_s ($\sigma = 0.15$)

$$x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} u(-4) \\ u(-3) \\ u(-2) \\ u(-1) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

The simulation result is shown in Fig. 4. The figure shows that both the states and input converge to zero by the proposed controller.

Next, we choose $\sigma = 0.1$ to study the design for the situation where σ is close to the lower bound of the feasible interval. In this case, the feasible solution for LMI is as follows.

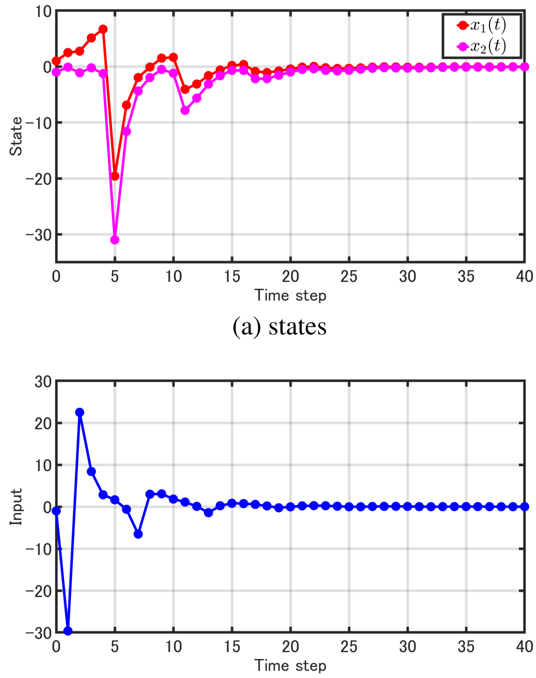
$$\mathcal{P} = \begin{bmatrix} 0.0902 & 0.1244 & -0.1286 & -0.0159 & 0.0030 & 0.0041 \\ 0.1244 & 0.1856 & -0.1639 & -0.0319 & -0.0053 & -0.0008 \\ -0.1286 & -0.1639 & 0.2345 & -0.0539 & -0.0107 & -0.0022 \\ -0.0159 & -0.0319 & -0.0539 & 0.1614 & -0.0559 & -0.0121 \\ 0.0030 & -0.0053 & -0.0107 & -0.0559 & 0.1354 & -0.0488 \\ 0.0041 & -0.0008 & -0.0022 & -0.0121 & -0.0488 & 0.1185 \end{bmatrix}$$

$$L = [\,0.0026 \ \ -0.0003 \ \ 0.0001 \ \ -0.0028 \ \ -0.0088 \ \ -0.0395\,]$$

$$\alpha = 0.0023, \quad \varepsilon = 0.0001$$

Then, the desired stabilizing feedback gain K is given by

$$K = \begin{bmatrix} -10.2783 & 0.5457 & -6.3120 & -4.0242 & -2.5294 & 1.5413 \end{bmatrix}$$



(a) states

(b) input

Fig. 4 Simulation result ($\sigma = 0.15$)

The closed-loop system $X(t+1) = \mathcal{A}X(t)$ is stabilized and the eigenvalues of $\mathcal{A}$ are

$$\{-0.4788, \ 0.9882, \ -0.2215 \pm 0.5069j, \ 0.4960 \pm 0.5026j\}$$

as shown in Fig. 5. It should be noted that there is an eigenvalue 0.9882 very close to the unit circle, which may cause degradation of the transient response of the closed-loop system.

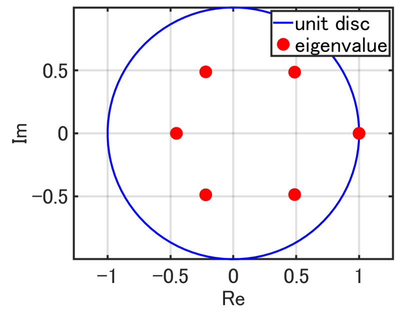
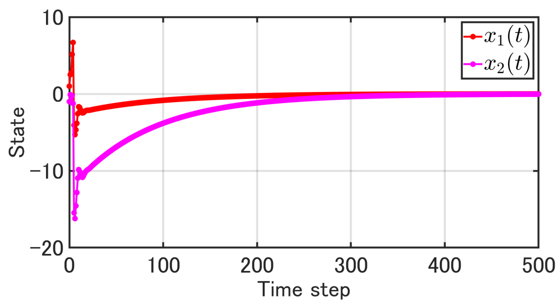


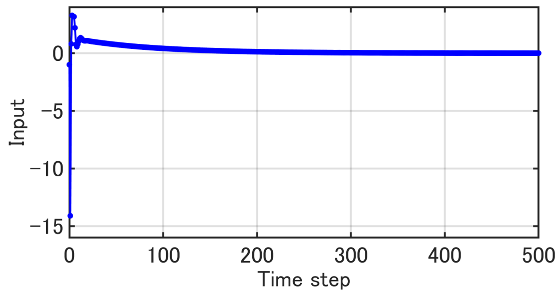
Fig. 5 Eigenvalues of closed-loop system for $A_s$and B_s ($\sigma = 0.15$)

We conduct a simulation for $\sigma = 0.1$ with the same initial values as the case of $\sigma = 0.15$. The simulation result is shown in Fig. 6. While the simulation for $\sigma = 0.1$ also resulted in stabilization, the convergence of the state and input trajectories was noticeably slower compared to the case of $\sigma = 0.15$. This indicates that the performance of the obtained controller strongly depends on the choice of σ.

From the above observations, we can conclude that the input-delay system is effectively stabilized with the augmented state feedback controller (6) obtained by the proposed data-driven design method without using the model (A, B).



(a) states



(b) input

Fig. 6 Simulation result ($\sigma = 0.1$)

5. CONCLUDING REMARKS

In this paper, we have derived the *data-driven* stability condition for input-delay systems based on the notion of *data informativity* [2]. Our proposed method enjoys the structure of input-delay systems and represents the system that fits the data compactly using the original (A, B) matrices, without resorting to an extended system. Moreover, it offers the advantage of efficiently designing the feedback gain through convex optimization. We have also validated the effectiveness of the proposed method by the numerical example.

As a direction of future works, we will extend the present results of this paper to practical data-driven control designs for linear systems with delays, such as tracking control, remote control with communication delays, and their experimental validations.

REFERENCES

[1] I. Markovsky and F. Dörfler: "Behavioral systems theory in data-driven analysis, signal processing, and control," *Annual Reviews in Control*, 52, 42/64 (2021)

[2] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel: "Data informativity: A new perspective on data-driven analysis and control," *IEEE Trans. on Automatic Control*, 65-21, 4753/4768 (2020)

[3] H. J. van Waarde, J. Eising, and M. K. Camlibel, and H. L. Trentelman: "The informativity approach: To data-driven analysis and control," *IEEE Control Systems Magazine*, 43-6, 32/66 (2023)

[4] H. J. van Waarde, M. K. Camlibel, and M. Mesbahi: "From Noisy Data to Feedback Controllers: Non-conservative Design via a Matrix S-Lemma," *IEEE Trans. on Automatic Control*, 67-1, 162/175 (2022)

[5] T. Fujinaka and M. Araki: "Discrete-time Optimal Control of Systems with Unilateral Time-delays," *Automatica*, 23, 763/765 (1987)

[6] J. Shirai, T. Yamaguchi, and K. Takaba: "Remote visual servo tracking control of drone taking account of time delays," *Proc. of 56th SICE Annual Conference*, 1589/1594 (2017)

[7] J.G. Rueda-Escobedo, E. Fridman and J. Schiffer: "Data-Driven Control for Linear Discrete-Time Delay Systems," *IEEE Trans. on Automatic Control*, 67-7, 3321/3336 (2022)