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Event-triggered Adaptive Control of a Drone-Ball-Beam System with Communication Failures

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Abstract: This paper proposes a novel event-triggered adaptive control of a drone-ball-beam (DBB) system, in which a drone and a ball-beam system are connected by wires. The proposed controller can track the ball position to a reference signal even under parameter variations. Furthermore, by introducing an integrator with an auxiliary signal into the control system, communication failures between the controller and the drone can be detected. The effectiveness of the proposed control system is evaluated by theoretical analysis and numerical simulation.

Keywords: Event-triggered control, adaptive control, drone, ball and beam system, communication failures.

1. INTRODUCTION

In recent years, works carried out by drones have become more complex and sophisticated. Drones are being used in various fields, such as rescue operations, automated transportation to remote islands, and so on.

Considering these circumstances, this paper addresses the problem of remotely controlling a ball-beam system [1, 2] by using a drone as shown in Fig. 1. This system is a drone and a ball-beam system connected by a wire called the drone-ball-beam (DBB) system. The ball's position is controlled by the angle of the beam, which the drone manipulates. Based on the event-triggered control [3] and high-gain adaptive control [4], this paper presents a new event-triggered adaptive control method for the DBB system. The proposed control system automatically adjusts controller parameters online, which enables robust control of ball mass variation. In addition, introducing the integrator with an auxiliary signal [5] to the control system makes it possible to detect failures in communication between the controller and the drone. After detection, switching to another communication line restores the stability and the control performance. The effectiveness of the proposed control system is verified not only by theoretical analysis but also by numerical simulation.

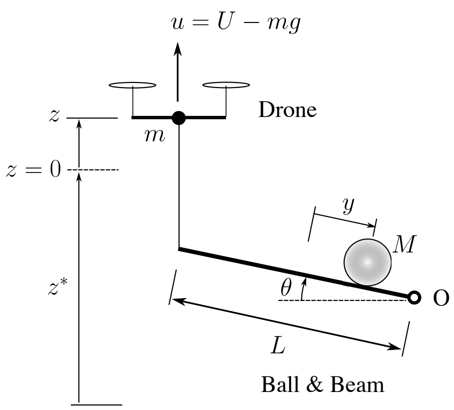

Fig. 1 A drone-ball-beam system

† Daiki Shinmura is the presenter of this paper.

Symbols

z	:	the vertical position of the drone
m	:	the mass of the drone
U	:	the thrust force of the drone
g	:	the acceleration of gravity
T	:	the tension of the wire
θ	:	the angle of the beam
J	:	the inertia of the beam (with the ball)
L	:	the length of the beam
y	:	the position of the ball from the center of the beam
M	:	the mass of the ball
b	:	the coefficient of the force by the gravity

Remark: z, θ and y are values from an equilibrium point, $(z, \theta, y) = (z^*, 0, 0)$. This paper uses the SI unit system.

2. PROBLEM STATEMENT

Consider a system which consists of a drone and ball-beam system as shown in Fig. 1. The drone pulls up the beam with a wire to control the position of the ball on the beam. The beam is free to rotate around "O". The mathematical models of those systems can be derived as follows [2].

$$m\ddot{z} = U - mg - T \tag{1}$$

$$J\ddot{\theta} = TL - Mg\left(\frac{L}{2} - y\right) \tag{2}$$

$$\ddot{y} = b\sin\theta = \frac{b}{L}z \tag{3}$$

It is well known that $b = 7$ if the approximation, $g \simeq 9.8$ is applied. This means that the model (3) does not depend on the sizes of the ball and the beams.

Now, define the error signal by

$$e := r - y \tag{4}$$

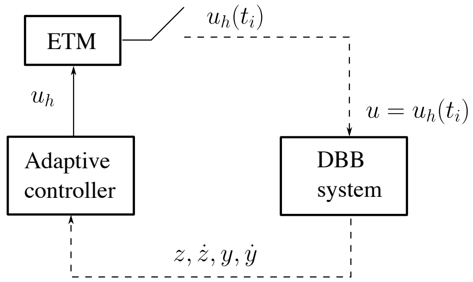

Fig. 2 The event-triggered adaptive control system

where r is the reference signal for y to track. The control objective is to achieve the following λ-tracking.

$$\limsup_{t\to\infty} |e(t)| \le \lambda \tag{5}$$

where $\lambda > 0$ is an arbitrarily small constant.

Assumption 1. z, $\dot{z}$, e, $\dot{e}$ $(= -\dot{y})$ and L are available.

3. CONTROL SYSTEM DESIGN

This section constructs the event-triggered adaptive control system shown in Fig. 2. The dashed lines indicate wireless communications, and "ETM" is the event-triggering mechanism.

3.1. Adaptive tracker by state feedback

First, the control input u is given by

$$u = U - mg \tag{6}$$

Then, the transfer function from u to y is calculated as

$$G(s) = \frac{Lb}{(mL^2 + J)s^4 - bMg} \tag{7}$$

Unfortunately, the following adaptive tracker cannot be applied to this system because its relative degree is four [4].

$$u_c = pe \tag{8}$$

$$\dot{p} = \sigma_0 e^2 - \sigma_1 p \tag{9}$$

where $\sigma_0 > 0$ and $\sigma_1 > 0$ are any constants. To solve this, introduce the derivative filter $H(s)$ to $G(s)$.

$$H(s) = s^3 + a_2 s^2 + a_1 s + a_0 \tag{10}$$

where $a_0 > 0$, $a_1 > 0$ and $a_3 > 0$ are constants selected so that $H(s)$ is a Hurwitz polynomial. Because the augmented system $G(s)H(s)$ has a relative degree of one, the adaptive tracking input u_c can be applied to the augmented system.

$$\begin{aligned} u &= \frac{d^3 u_c}{dt^3} + a_2 \frac{d^2 u_c}{dt^2} + a_1 \frac{du_c}{dt} + a_0 u_c \\ &= f_3(e, \dot{e}, z, \dot{z}, p) + a_2 f_2(e, \dot{e}, z, p) \\ &\quad + a_1 f_1(e, \dot{e}, p) + a_0(pe) \\ &=: u_h \end{aligned} \tag{11}$$

where f_1, f_2 and f_3 are shown in Appendix.

3.2. Event-triggering mechanism

Next, introduce the ETM [3] on the controller side. The control input u can be rewritten as follows.

$$u(t) = u_h(t_i),\ \forall t \in [t_i, t_{i+1}) \tag{12}$$

Each t_i $(i = 0, 1, \ldots)$ is defined by the triggering law: $t_0 = 0$ and

$$t_{i+1} := \inf\{t > t_i \mid |\varepsilon(t)| > \varepsilon_0\} \tag{13}$$

$$\varepsilon := u_h(t_i) - u_h(t),\ \forall t \in [t_i, t_{i+1}) \tag{14}$$

where $\varepsilon_0 > 0$ is a constant. Using the above ETM guarantees that $|\varepsilon(t)| \le \varepsilon_0, \forall t \ge 0$.

Lemma 1. All the signals of the event-triggered adaptive control system constructed above are bounded. Moreover, for given λ, there exist γ, σ and T such that the λ-tracking (5) can be achieved.

Proof. The transformed state equation of the augmented system is expressed as

$$\begin{aligned} \dot{y} &= \alpha y + \beta(u_c + \tilde{\varepsilon}) + \varphi^\top w + l_0,\ \beta > 0 \\ \dot{w} &= \Phi w + \kappa y + l_1 \end{aligned} \tag{15}$$

where α, β, φ and Φ are constants, a vector and a matrix respectively, w is the state of the augmented system, $\tilde{\varepsilon}$ is a virtual signal of ε filtered by $1/H(s)$, and l_0 and l_1 are bounded disturbances caused by the gravity.

The error system can be obtained as follows.

$$\begin{aligned} \dot{e} &= -(\beta p^* - \alpha)e + \beta(p^* - p)e \\ &\quad -\beta\tilde{\varepsilon} - \varphi^\top w - l_0, \\ \dot{w} &= \Phi w + \kappa r - \kappa e - l_1 \end{aligned} \tag{16}$$

Because of the minimum phase property of (14), all eigenvalues of Φ lie in the left half complex plane. Hence, for any positive definite matrix Q, there exists a positive definite matrix P such that $\Phi^\top P + P\Phi = -2Q$.

Define a positive definite function by

$$V = \frac{1}{2}\left\{e^2 + w^\top P w + \frac{1}{\sigma_0}(p^* - p)^2\right\} \tag{17}$$

The time derivative of V can be evaluated by

$$\begin{aligned} \dot{V} \le &-\frac{1}{2}\underbrace{\left(2\beta p^* - 2\alpha - \frac{\beta^2}{\delta} - \|\varphi\|^2 - \kappa^2 - 1\right)}_{\rho_1} e^2 \\ &-\frac{1}{2}\underbrace{\left(2\lambda_{\min}(Q) - 3 - \frac{\kappa^2}{\delta}\right)}_{\rho_2}\|w\|^2 \\ &-\frac{1}{2\sigma_0}\underbrace{\left(\sigma_1 - \frac{{\sigma_1}^2(p^*)^2}{\delta}\right)}_{\rho_3}(p^* - p)^2 \\ &+\frac{\delta}{2}\underbrace{\left(\bar{\varepsilon}_0^2 + l^2 + r^2 + 1\right)}_{\rho_0} \end{aligned} \tag{18}$$

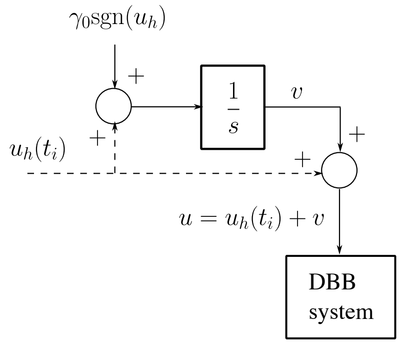

Fig. 3 Introduction of the integrator

where $\tilde{\varepsilon}_0 > \tilde{\varepsilon}(t),\ l > \max\{|l_0(t)|,\ |l_1(t)|\}, \forall t \geq 0$.

For given small δ, Q exists so that $\rho_2 > 0$, and there exist sufficiently small σ_1, large σ_0 and p^* such that $\rho_1 > 0$ and $\rho_3 > 0$. Thus, the following inequality holds.

$$\dot{V} \leq -\rho V + \frac{\delta \rho_0}{2},\ \rho = \min\left\{\rho_1, \frac{\rho_2}{\lambda_{\max}(P)}, \rho_3\right\} \quad (19)$$

Therefore, the signals, e, w and p are bounded. Because $|e| \leq \sqrt{2V}$, it is shown that

$$\limsup_{t\to\infty} |e(t)| \leq \sqrt{\frac{\delta\rho_0}{\rho}} \quad (20)$$

Taking δ so that $\lambda > \delta\rho_0/\rho$ ensures (5).

Thus, the proof is completed. ■

4. FAILURE DETECTION

If communication is lost, the drone will no longer be able to receive the input signal. Hence, the model of the communication failure is expressed by

$$u(t) = \xi,\ \forall t \geq t_F \quad (21)$$

where ξ is an unknown constant, and $t_F > 0$ is an unknown time when the failure occurs.

To detect the failure (21), an integrator is introduced as shown in Fig. 3 Then, the control input is modified as follows [5].

$$u(t) = u_h(t_i) + v(t),\ \forall t \in [t_i, t_{i+1}) \quad (22)$$
$$\dot{v}(t) = u_h(t_i) + \gamma_0 \mathrm{sgn}(u_h(t_i)) \quad (23)$$

where $\gamma_0 > 0$ is a constant.

When the failure (21) happens, v obeys

$$\dot{v} = \xi + \gamma_0 \mathrm{sgn}(\xi) \quad (24)$$

which yields

$$\begin{cases} \dot{v} > \gamma_0 & (\xi \geq 0) \\ \dot{v} < -\gamma_0 & (\xi < 0) \end{cases} \quad (25)$$

Thus $|v|$ tends to diverge if the failure occurs. By using this property, the detection can be achieved. That is, the detection time is defined by

$$t_D := \inf\{t \mid |v(t)| > \Gamma\} \quad (26)$$

After the detection, the drone switches the failed the communication link to another link to restore the stability.

Then, the following main results of this paper can be obtained.

Theorem 1. The event-triggered adaptive control system constructed above has the following properties.
(P1) If the failure (21) occurs, then, the detection time t_D exists and satisfies

$$t_D - t_F < \frac{2\Gamma}{\gamma_0} \quad (27)$$

(P2) All the signals are bounded before and after the failure. Moreover, the λ-tracking (5) can be achieved.

Proof. Consider the case when the failure occurs. From (28), it follows that

$$v(t) > v(t_F) + \gamma_0(t - t_F)\ \ (\xi \geq 0) \quad (28)$$
$$v(t) < v(t_F) - \gamma_0(t - t_F)\ \ (\xi < 0) \quad (29)$$

Therefore, t_D defined by (26) exists and satisfies

$$t_D - t_F < \frac{\Gamma + |v(t_F)|}{\gamma_0} \quad (30)$$

Because of $|v(t_F)| < \Gamma$, the inequality (27) holds. (P1) is true.

From (22), the transfer function of the augmented system including the integrator can be represented as

$$G(s)\left(1 + \frac{1}{s}\right)H(s) = \frac{Lb(s+1)(s^3 + a_2 s^2 + a_1 s + a_0)}{(mL^2 + J)s^5 - bMgs} \quad (31)$$

This system is a minimum-phase and has the relative degree of one. Therefore, the adaptive controller (8) can be applied and thus Lemma1 holds for the system (31). The system does not have a finite explosion time, and the failure can be detected within a finite time. Thus, all the signals in the control system are bounded before and after the failure. That is, (P2) is true. ■

5. SIMULATION

To confirm the effectiveness of the proposed control system, numerical simulation are shown.

The parameters pf the DBB system are shown below.

$$m = 0.1 \text{ [kg]},\ L = 1 \text{ [m]},\ M = 0.02 \text{ [kg]}$$

The parameters of the controller are selected as follows.

$$\sigma_0 = 10,\ \sigma_1 = 0.01,\ p(0) = 0.01,\ \varepsilon_0 = 0.002,$$
$$\gamma_0 = 0.1,\ \Gamma = 0.1$$

The simulation results are shown in Fig. 4. The reference signal of the ball is set as $r = 0.3$ [m]. The communication failure occurs at $t_F = 20$ [s], and the detection time is $t_D \simeq 20.60$ [s] (see the vertical red lines). From (a) to (d), all the signals are bounded, and the tracking control can be achieved before and after the failure. Because the number of communication does not increase at steady state, the effect of the ETM is confirmed (see (e)).

In this simulation, a centrifugal force term is added to the model (3). The results show that the control system has robustness with respect to nonlinearity of the centrifugal force.

6. CONCLUSION

This paper presents the event-triggered adaptive control of the DBB system with communication failures. The effectiveness of the proposed control system has been confirmed by theoretical analysis and numerical simulation.

In future works, the experiments by using an actual DBB system will be explored. Additionally, robustness against external disturbances such as wind will be taken into consideration.

APPENDIX

In (11), f_1, f_2 and f_3 are given as follows.

$$f_1 = e(\sigma_0 e^2 - \sigma_1 p) + p\dot{e}$$

$$f_2 = \frac{df_1}{dt} = \frac{\partial f_1}{\partial e}\dot{e} + \frac{\partial f_1}{\partial \dot{e}}\left(-\frac{b}{L}z\right) + \frac{\partial f_1}{\partial p}\left(\sigma_0 e^2 - \sigma_1 p\right)$$

$$f_3 = \frac{df_2}{dt} = \frac{\partial f_2}{\partial e}\dot{e} + \frac{\partial f_2}{\partial \dot{e}}\left(-\frac{b}{L}z\right) + \frac{\partial f_2}{\partial p}\left(\sigma_0 e^2 - \sigma_1 p\right) + \frac{\partial f_2}{\partial z}\dot{z}$$

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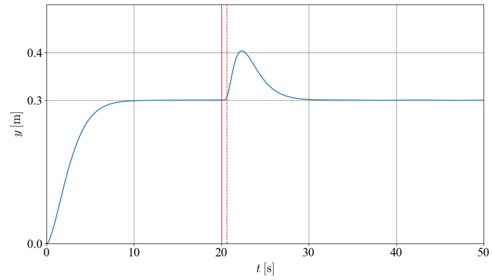
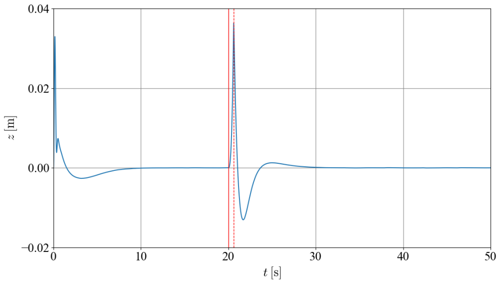
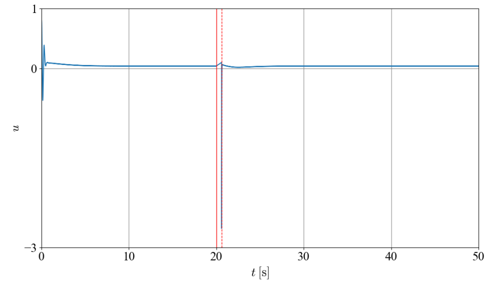


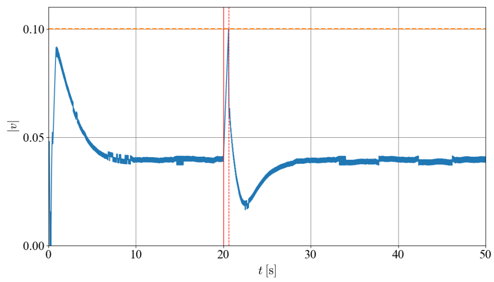
(a) The position of the ball, y [m]



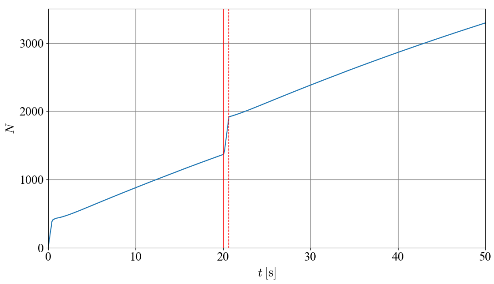
(b) The position of the drone, z [m]



(c) The control input, u



(d) The absolute value of the integrator output, v



(e) The number of communication, N

Fig. 4 The event-triggered adaptive control system