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A Hybrid Approach to Switching Observer under Disturbance and Measurement Dropout

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Abstract: Two switching observer (SO) schemes have been proposed recently in order to address unknown disturbance and measurement dropout. Each of them has its benifit and drawback, depending on conditions of the signal loss pattern, disturbance dynamics, and system matrices. This paper discusses these drawbacks and whether or not they can be remedied by a hybrid approach taking advantage of each.

Keywords: Switching observer, Signal loss, Unknown disturbance, Linear matrix inequality.

1. INTRODUCTION

Networked control/sensing has been attracting much attention recently [1]. In this paper we consider state estimation of a dynamical system under disturbance and measurement dropout. The system to be estimated is in the remote side and subject to an unknown disturbance, while in the local side we receive multiple observation suffering from irregular signal (packet) loss due to transmission error and sensing failure (say, due to occlusion in a camera sensor). Fig. 1 shows an image of disturbance and irregular measurement dropout.

In such a difficult situation the author's group have proposed various types of switching observers (SO) [2-4]. In this paper we discuss drawbacks of these schemes and aim to remedy them via a hybrid approach.

In what follows, the i-th component of a vector y is denoted by y_i without explicit definition. For a finite set π of indices $1, 2, \cdots, p$, we denote by $\#\pi$ the number of its elements. If $\#\pi = 1$, we may write $\pi = i$ instead of $\pi = \{i\}$ for brevity, with a slight abuse.

2. PROBLEM FORMULATION

Consider that a linear discrete-time system

$$\begin{cases} x[k+1] = Ax[k] + B_d d[k], \\ y[k] = Cx[k] \end{cases} \qquad k = 0, 1, 2, \cdots \quad (1)$$

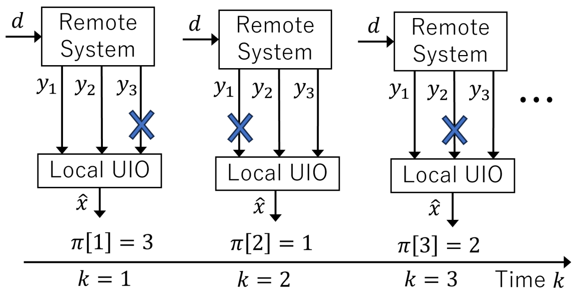

Fig. 1 Image of unknown disturbance and lossy multiple measurement.

† Kenji Sugimoto is the presenter of this paper.

Table 1 Example of signal loss pattern.

k	1	2	3	4	5	6	7	8	9	...
$y_1[k]$	S	F	S	S	S	S	F	S	F	...
$y_2[k]$	S	S	F	S	S	S	S	F	F	...
$y_3[k]$	F	S	S	F	S	S	F	S	S	...
$\pi[k]$	3	1	2	3	ϕ	ϕ	{1,3}	2	{1,2}	...

is in the remote side over lossy multiple communication channels, where $x \in \mathbb{R}^n, y \in \mathbb{R}^p$, and $d \in \mathbb{R}^m$ are respectively the state, the output, and the unknown disturbance. We assume $n \geq p > m \geq 1$, hence Eq. (1) is a multi-output and tall system. A, B_d, and C are constant matrices of compatible sizes. We assume that (C, A) is observable and B_d has full column rank.

Our objective is to construct a state observer without recourse to d, when some components of y may be lost irregularly. We assume that in the local side we can detect which component $y_i[k], i = 1, \cdots, p$ is lost at time step k, in real time. We define the set $\pi[k]$ of all such loss channel indices. If no loss at k, then $\pi[k] = \phi$. Table 1 shows an example of the signal loss pattern, where S means "received successfully" and F means "receipt failure."

Let us define an output matrix corresponding to "success" channels.

Definition: For any $\pi \subset \{1, 2, \cdots, p\}$, we define by $\tilde{C}_\pi$ the matrix made by removing all rows c_i of C such that $i \in \pi$.

If, for example, $p = 3$ and

$$C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \qquad c_i \in \mathbb{R}^{1\times n},\ i = 1, 2, 3,$$

then we have $\tilde{C}_\phi = C$, and

$$\tilde{C}_1 = \begin{pmatrix} c_2 \\ c_3 \end{pmatrix},\ \tilde{C}_2 = \begin{pmatrix} c_1 \\ c_3 \end{pmatrix},\ \tilde{C}_3 = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix},$$
$$\tilde{C}_{\{2,3\}} = c_1,\ \tilde{C}_{\{1,3\}} = c_2,\ \tilde{C}_{\{1,2\}} = c_3.$$

Definition: For each k we receive $\tilde{y}_{\pi[k]}[k] = \tilde{C}_{\pi[k]}x[k]$ in Eq. (1) with the detected set $\pi[k]$ of loss channel indices while in operation. But $\pi[k]$ is unavailable in advance, so that we need to design SO that maintains stability for any signal loss pattern.

We can obviously construct a standard observer if the set $\pi[k] \equiv \pi$ is invariant with respect to k, $(A, \tilde{C}_\pi)$ is observable, and $d[k]$ is available for all k in the local side. In contrast, we assume in this paper that $\pi[k]$ changes irregularly with k and that $d[k]$ is unknown.

3. RELATED WORKS

Assume for a moment that there is no measurement dropout. In this case, a disturbance observer (DO) approach [5-7] is known to be effective under the assumption that a generation model of the disturbance is available. If, in particular,

$$d[k+1] = d[k], \quad k = 0, 1, 2, \cdots \tag{2}$$

and

$$\text{rank} \begin{pmatrix} A - I & B_d \\ C & 0 \end{pmatrix} = n + m, \tag{3}$$

then DO is given by

$$\begin{pmatrix} \hat{d}[k+1] \\ \hat{x}[k+1] \end{pmatrix} = \begin{pmatrix} I & 0 \\ B_d & A \end{pmatrix} \begin{pmatrix} \hat{d}[k] \\ \hat{x}[k] \end{pmatrix} + \begin{pmatrix} L_d \\ L_x \end{pmatrix} (y[k] - C\hat{x}[k]), \quad k = 0, 1, 2, \cdots \tag{4}$$

for gains L_d, L_x that stabilize

$$\begin{pmatrix} I & -L_d C \\ B_d & A - L_x C \end{pmatrix}.$$

Another approach to address the disturbance is unknown input observer (UIO) [8-10]. If we assume that

$$\text{rank}\, CB_d = m; \tag{5}$$

and

$$\text{rank} \begin{pmatrix} A - \lambda I & B_d \\ C & 0 \end{pmatrix} = n + m \quad \forall \lambda \in \mathbb{C}, \tag{6}$$

then UIO is represented as

$$\hat{x}[k+1] = (A - LC - MCA)\hat{x}[k] + Ly[k] + My[k+1], \quad k = 0, 1, 2, \cdots \tag{7}$$

for gains

$$M = B_d(CB_d)^\dagger,$$

and L that stabilizes $A - MCA - LC$, where † means the pseudo inverse matrix. There exists such L, since the pair $(A - MCA, C)$ is observable if and only if Eq. (6) holds; see [8] for a proof.

Now we go back to our lossy measurement case. By making use of $\tilde{C}_{\pi[k]}$ defined in Section 2, a gain-switching DO [2] has been proposed to deal with irregular signal loss. Furthermore, a gain-switching UIO [3] has also been proposed to address the problem of signal loss under the assumption that $\#\pi[k] \leq 1$. In the both cases a simultaneous solution of linear matrix inequalities (LMIs) plays a key role to design switching gains that stabilize the error system under any signal loss pattern when the above assumptions hold. We omit details in order to avoid overlap but it is worth pointing out that, by introducing a new state vector ξ, we can transform Eq. (7) to

$$\hat{x}[k] = \xi[k] + M(y[k] - CA\hat{x}[k-1]),$$
$$\xi[k+1] = A\hat{x}[k] + L(y[k] - C\hat{x}[k]),$$

which is more convenient for addressing signal loss.

4. DISCUSSION

Fig. 2 is a block diagram of gain-switching DO to address signal loss [2]. A drawback of this scheme is that it assumes a generation model of the disturbance. In Eq. (4), for example, we have assumed $d[k] \equiv d$ hence $\hat{x}[k] - x[k]$ does not converge to zero as $k \to \infty$ if $d[k]$ varies with k. The scheme may end up with a poor response even if $d[k]$ is stepwise constant.

Fig. 3 is, on the other hand, a block diagram of gain-switching UIO to address signal loss [3]. A drawback of this scheme is the assumption $\#\pi[k] \leq 1$. This condition may sometimes fail in practice since sensing failure or transmission error takes place randomly. Once this condition fails, we can no longer continue the state estimation in a stable way.

In view of these, it is natural to expect that a hybrid approach may be effective by taking advantages of the above two observers. One such approach would be a re-

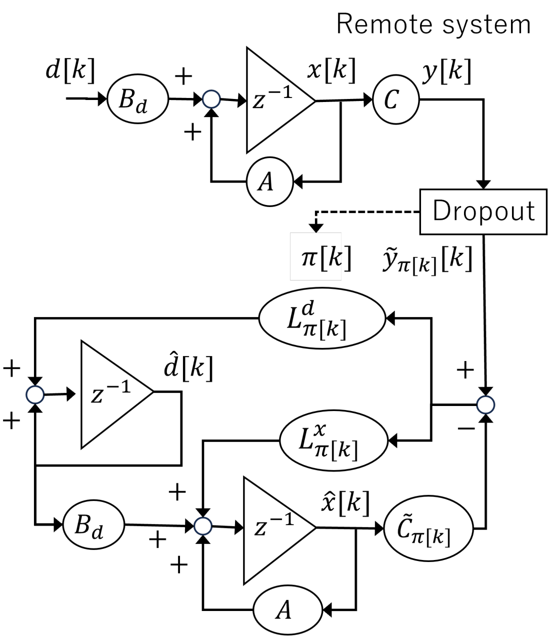

Fig. 2 Gain-switching DO.

newal law:

$$\hat{x}[k] = \begin{cases} \xi[k] + M_{\pi[k]}(\tilde{y}_{\pi[k]} - \tilde{C}_{\pi[k]}A\hat{x}[k-1]), & \text{if } \#\pi[k] \le 1 \\ \xi[k] + B_d\hat{d}[k-1], & \text{if } \#\pi[k] > 1 \end{cases}$$

$$\xi[k+1] = A\hat{x}[k] + L^x_{\pi[k]}(\tilde{y}_{\pi[k]} - \tilde{C}_{\pi[k]}\hat{x}[k]),$$
$$\hat{d}[k+1] = \hat{d}[k] + L^d_{\pi[k]}(\tilde{y}_{\pi[k]} - \tilde{C}_{\pi[k]}\hat{x}[k]),$$

for $k = 0, 1, 2, \cdots$.

The objective of the present paper is to discuss how to design the gains L^x_i, L^d_i, and M_j for all possible i's and j's that stabilize the above hybrid switching system, and evaluate its pros and cons in comparison with existing schemes.

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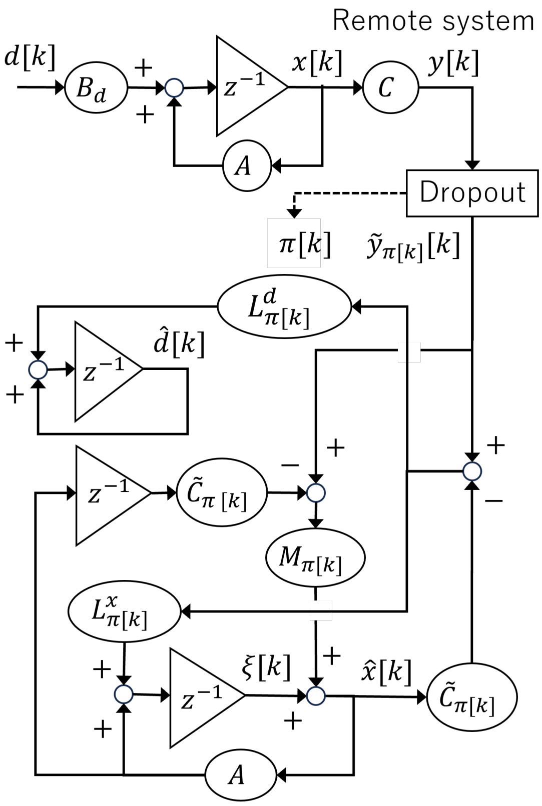

Fig. 3 Gain-switching UIO.

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