Distributed non-fragile $H_{\infty}$ filtering of discrete-time systems with randomly occurring gain variations

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Abstract: The issue of distributed non-fragile $H_{\infty}$ filtering for discrete-time systems with randomly occurring gain variations over sensor networks is investigated in this paper. The randomly occurring filter gain variations obey a known probability distribution. Based on stochastic analysis and Lyapunov function approach, the sufficient condition is presented to guarantee the desired stochastic stability and $H_{\infty}$ disturbance attenuation level. The solution of the desired distributed filter gains is characterized by solving a set of linear matrix inequalities. Finally, a numerical simulation example is provided to show the effectiveness of the proposed design approach.

Keywords: Distributed $H_{\infty}$ filtering, non-fragile filtering, sensor network, randomly occurring gain variations.

1. INTRODUCTION

Sensor networks are composed of sensor nodes with sensing, computation, and wireless communication abilities. Compared with the common attributes of complex networks, sensor networks have their own characteristics because of the large number of the wireless nodes distributed in the space. In the past decades, sensor networks have been applied widely in many fields [1-4].

Recently, the problem of distributed state estimation over sensor networks has been investigated for discrete-time systems with time-varying delay and randomly varying nonlinearities [5]. Considering the occurrence of signal attenuation and limited bandwidth due to the communication capacity constraint in networked control systems [6], the finite-time distributed state estimation problem for discrete-time nonlinear systems over sensor networks has been discussed in [7]. The Round-Robin protocol has been introduced to overcome the channel capacity constraint and the fading model has been employed to represent the unreliable communication channels [7]. In [8, 9], distributed estimation and filtering over sensor networks subject to sensor saturations have been studied. Over the past few years, as a fundamental problem in theoretical studies and applications over sensor networks, distributed $H_{\infty}$ filtering has gained a large number of interest [10-15].

As pointed out in [16] and [17], there do exist uncertainties in the controller/filter due to some unexpected errors during the implementation, owing to the numerical roundoff errors, limited word length of the computer, etc. And under this circumstance, the stability, performance and reliability of the systems will be influenced [18-20]. As a result, the non-fragile control/filtering has been proposed to deal with the uncertainties of such controllers/filters. In other words, the capability of adapting the variations of the controllers/filters is so-called non-fragility. For example, in [21], a new gain-scheduling approach to non-fragile $H_{\infty}$ fuzzy control subject to fading channels has been investigated. And [22] has dealt with the problem of the robust non-fragile filtering for uncertain linear systems with estimator gain uncertainties.

However, the occurrence of filter gain variations may suffer from the random changes of environment in a networked situation. In this case, the gain variations may be in a probabilistic way. Therefore, the phenomenon of randomly occurring gain variations (ROGVs) should be taken into account when designing filters. In [23], the problem of non-fragile $H_{\infty}$ fuzzy filtering with ROGVs and channel fading has been studied. A non-fragile $H_{\infty}$ filter which can tolerate ROGVs and channel fading has been designed for a class of nonlinear systems in [24]. However, distributed non-fragile $H_{\infty}$ filtering over sensor networks for discrete-time systems with ROGVs has not been investigated adequately. Therefore, it has vital practical significance to consider the distributed non-fragile $H_{\infty}$ filtering problem with ROGVs which is the motivation of this paper.

In this paper, we focus on the problem of distributed non-fragile $H_{\infty}$ filtering for discrete-time systems with randomly occurring gain variations over sensor networks. First, we utilize a stochastic variable with a known probability distribution to represent the probabilistic property of randomly occurring gain variations. Second, by means of stochastic analysis method, the $H_{\infty}$ performance criterion is established for the system under consideration. Then, a distributed non-fragile $H_{\infty}$ filter which can solve the sparsity problem is designed. Finally, a numerical example and its simulation is provided to illustrate the correctness of the method.

Notations: The notations used in this paper are standard. The Kronecker product of matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{p \times q}$ is denoted as $A \otimes B$. $E\{\alpha\}$, $E\{\alpha|\beta\}$ mean, respectively, the mathematical expectation of the stochastic variable $\alpha$ and the expectation of $\alpha$ conditional on $\beta$. $\lambda_{\min}(A)$ represents the minimal eigenvalue of matrix $A$. 

$^\dagger$ Yun Chen is the presenter of this paper.

This work was supported in part by the National Natural Science Foundation of China under grants U1509205 and 61473107, and in part by the Zhejiang Provincial Natural Science Foundation of China under grant LR16F030003. The presenter of this paper.
2. PROBLEM FORMULATION AND PRELIMINARIES

The sensor network has \( N \) sensor nodes with a specific network topology represented by a directed graph \( G = (N,E,C) \), where \( N = \{1,2,\ldots,N\} \) is the set of sensor nodes; \( E \subseteq N \times N \) denotes the set of edges; \( C = [c_{ij}]_{N \times N} \) is the weighted adjacency matrix with adjacency element \( c_{ij} \). \( c_{ij} > 0 \) if edge \((i,j) \in E\) means that there is a signal transmission from sensor \( j \) to sensor \( i \). If \( i = j \), we denote \( c_{ii} = 1 \) for all \( i \in N \), that is, the sensor set is self-connected. The set of neighbors of node \( i \in N \) and the node itself can be denoted by \( N_i = \{ j \in N \mid (i,j) \in E \} \).

Consider this following discrete-time system:

\[
\begin{align*}
\dot{x}(k+1) &= A x(k) + B v(k) \\
y_j(k) &= C_j x(k) + D_j v(k) \\
z(k) &= H x(k)
\end{align*}
\]  
(1)

where \( x(k) \in \mathbb{R}^n \) is the state vector of target plant; \( y_j(k) \in \mathbb{R}^{n_j} \), \( j \in \{1,2,\ldots,N\} \) is the measurement output measured by the \( j \)-th sensor; \( z(k) \in \mathbb{R}^n \) is the output signal to be estimated; \( v(k) \in \mathbb{R}^m \) is the exogenous disturbance which belongs to \( L_2[0,\infty) \); \( A, B, C_j, D_j, H \) are known real matrices with appropriate dimensions.

Now, we consider the following filter:

\[
\begin{align*}
\dot{\hat{x}}_i(k+1) &= A \hat{x}_i(k) + \sum_{j \in N_i} c_{ij}(L_{ij} + \alpha(k) M_{ij}) \bar{y}_j(k) \\
\dot{\bar{z}}_i(k) &= H \hat{x}_i(k)
\end{align*}
\]  
(2)

where \( \hat{x}_i(k) \in \mathbb{R}^{n_i} \) is the estimate of \( x(k) \) from the \( i \)-th sensor node; \( \bar{z}_i(k) \in \mathbb{R}^{n_i} \) is the vector to be estimated from the \( i \)-th filter node; \( L_{ij} \) is the filter gain matrix to be designed.

In this paper, we utilize \( \alpha(k) \) to govern the probabilistic property of the randomly occurring gain variations, where \( \alpha(k) \in [\alpha_1, \alpha_2] \) is a stochastic variable obeying any known stochastic distribution with \( E\{\alpha(k)\} = \tilde{\alpha} \) and \( E\{(\alpha(k) - \tilde{\alpha})^2\} = \sigma^2 \), with \( \alpha_1, \alpha_2, \sigma^2 \) being known constants. \( M_{ij} \) are known matrices with appropriate dimensions which indicates the filter gain variations.

**Remark 2.1** The parameters of the distributed filters over sensor networks may be subject to random changes during the implementation in a networked environment. Such random changes result from a variety of reasons such as network-induced random failures and sudden environmental disturbances, etc. Different from [25], in this paper, we aim to design a distributed non-fragile \( H_\infty \) filter (2) for the discrete-time system (1), where the ROGVs are described by \( \alpha(k) M_{ij} \).

Setting \( \tilde{x}_i(k) = x(k) - \hat{x}_i(k) \), the estimation error can be easily obtained from (1) and (2) as follows:

\[
\begin{align*}
\tilde{x}_i(k+1) &= A \tilde{x}_i(k) - \sum_{j \in N_i} c_{ij} L_{ij} C_j \tilde{y}_j(k) \\
&\quad - (\alpha(k) - \tilde{\alpha}) \sum_{j \in N_i} c_{ij} M_{ij} C_j \tilde{y}_j(k) \\
&\quad - \tilde{\alpha} \sum_{j \in N_i} c_{ij} M_{ij} \tilde{y}_j(k) \\
&\quad + \{B - \sum_{j \in N_i} c_{ij} L_{ij} D_j \} \bar{v}(k) \\
&\quad + (\alpha(k) - \tilde{\alpha}) \sum_{j \in N_i} c_{ij} M_{ij} D_j \bar{v}(k) \\
&\quad - \tilde{\alpha} \sum_{j \in N_i} c_{ij} M_{ij} D_j \bar{v}(k) \\
&\quad + \{B - \sum_{j \in N_i} c_{ij} L_{ij} D_j \} \bar{v}(k) \\
&\quad + (\alpha(k) - \tilde{\alpha}) \sum_{j \in N_i} c_{ij} M_{ij} D_j \bar{v}(k) \\
&\quad - \tilde{\alpha} \sum_{j \in N_i} c_{ij} M_{ij} D_j \bar{v}(k)
\end{align*}
\]  
(3)

\[
\begin{align*}
\tilde{z}_i(k) &= \bar{z}(k) - \tilde{z}_i(k) = H \tilde{x}_i(k).
\end{align*}
\]

By using the Kronecker product, the error dynamics can be rewritten as

\[
\begin{align*}
\tilde{x}(k+1) &= [ I_N \otimes A - L \tilde{C} - (\alpha(k) - \tilde{\alpha}) L_M \tilde{C} \\
&\quad - \tilde{\alpha} L_M \tilde{C} ] \tilde{x}(k) + \{B - LD \} \bar{v}(k) \\
&\quad - (\alpha(k) - \tilde{\alpha}) L_M \tilde{D} - \tilde{\alpha} L_M \tilde{D} \} \bar{v}(k) \\
&\quad + (\alpha(k) - \tilde{\alpha}) L_M \tilde{D} \bar{v}(k) \\
&\quad - \tilde{\alpha} L_M \tilde{D} \bar{v}(k)
\end{align*}
\]  
(4)

\[
\begin{align*}
\tilde{z}(k) &= \tilde{H} \tilde{x}(k)
\end{align*}
\]

where

\[
\begin{align*}
\tilde{x}(k) &= (\bar{x}_1^T, \bar{x}_2^T, \ldots, \bar{x}_N^T)^T \\
\tilde{z}(k) &= (\tilde{z}_1^T, \tilde{z}_2^T, \ldots, \tilde{z}_N^T)^T \\
\tilde{L} &= [c_{ij} L_{ij}]_{N \times N} \\
\tilde{C} &= \text{diag}\{C_1, C_2, \ldots, C_N\} \\
\tilde{D} &= [D_1^T, D_2^T, \ldots, D_N^T]^T \\
\tilde{B} &= [B_1^T, B_2^T, \ldots, B_N^T]^T \\
\tilde{H} &= \text{diag}\{H, H, \ldots, H\}
\end{align*}
\]

It should be mentioned that \( \tilde{L} = [c_{ij} L_{ij}]_{N \times N} \) is a sparse matrix, where \( \tilde{L} \in W_{n_x \times n_x} \), which is defined as:

\[
\begin{align*}
W_{n_x \times n_x} &= \{ W = [W_{ij}] \in \mathbb{R}^{n_x \times n_x} \mid W_{ij} \in \mathbb{R}^{n_x \times n_x}, \\
&\quad W_{ij} = 0, \forall j \notin \{N\} \}.
\end{align*}
\]  
(5)

By defining \( \eta(k) = [x^T(k), \tilde{x}^T(k)]^T \) and \( \bar{z}(k) = [\tilde{z}^T(k), \tilde{z}^T(k)]^T \), we can obtain the following augmented system:

\[
\begin{align*}
\eta(k+1) &= (A - C) \eta(k) + (B_1 - B_2) \bar{v}(k) \\
\bar{z}(k) &= H \eta(k)
\end{align*}
\]  
(6)

where

\[
\begin{align*}
A &= \begin{bmatrix} A & 0 \\ 0 & \Theta_1 \end{bmatrix}, \\
C &= \begin{bmatrix} 0 & 0 \\ 0 & \Theta_2 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} B \\ \Theta_3 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0 \\ \Theta_4 \end{bmatrix}, \\
H &= \begin{bmatrix} H & 0 \\ 0 & \tilde{H} \end{bmatrix}
\end{align*}
\]
To this end, we present the following definition for the augmented system (6).

**Definition 2.1** Given a scalar \( \gamma > 0 \), the filter (2) is said to be a stochastic \( H_\infty \) filter of system (1) if system (6) with \( v(k) \equiv 0 \) is stochastically stable and under zero initial condition, for all nonzero \( v(k) \), system (6) has a certain prescribed \( H_\infty \) disturbance attenuation level \( \gamma > 0 \) satisfying

\[
\sum_{k=0}^{\infty} E[\|x(k)\|^2] \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2.
\]

The main purpose of this paper is to design a distributed non-fragile \( H_\infty \) filter (2) for system (1) with randomly occurring gain variations such that the augmented system (6) satisfies the \( H_\infty \) performance.

## 3. MAIN RESULTS

In this section, by utilizing Lyapunov function approach and stochastic analysis technique, we will provide the design method of distribute non-fragile \( H_\infty \) filter (2) for system (1). We now start by presenting the following lemma that will be used in the derivations.

**Lemma 3.1** Let \( \mathbf{U} = \text{diag}\{U_{11}, U_{22}, \cdots, U_{NN}\} \), with \( U_{ii} \in \mathbb{R}^{n_i \times n_i} \) \( (i = 1, 2, \cdots, N) \) being invertible matrices. If \( \mathbf{X} = \mathbf{W} \mathbf{U} \) for \( \mathbf{W} \in \mathbb{R}^{m \times n} \), then we have \( \mathbf{W} \in \mathbb{W}_m(n_i, n_j) \leftrightarrow \mathbf{X} \in \mathbb{W}_m(n_i, n_j) \).

Now, we propose the following analysis criterion which is the theoretical basis for the filter design.

**Theorem 3.1** Consider the discrete-time system (1). For a given \( \mathbf{L} \) composed by filter gain matrices \( L_{ij}(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N) \) and a prescribed disturbance attenuation level \( \gamma > 0 \), the augmented system (6) satisfies the \( H_\infty \) performance constraint (7), if there exist matrices \( P_j > 0(j = 1, 2, \cdots, N+1) \), \( R \) such that

\[
\mathbf{P} = \begin{bmatrix} P_1 & R \\ * & P_2 \end{bmatrix} > 0,
\]

and

\[
\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ * & \tilde{A}_{22} \end{bmatrix} < 0
\]

where

\[
\tilde{A}_{11} = A^T \mathbf{P} \mathbf{A} - \dot{\mathbf{C}}^T \dot{\mathbf{P}} \mathbf{C} + \mathbf{H}^T \mathbf{H},
\]

\[
\tilde{A}_{12} = A^T \mathbf{P} \mathbf{B}_1 + \dot{\mathbf{C}}^T \dot{\mathbf{P}} \mathbf{B}_2,
\]

\[
\tilde{A}_{22} = \mathbf{B}_1^T \mathbf{P} \mathbf{B}_1 + \mathbf{B}_2^T \mathbf{P} \mathbf{B}_2 - \gamma^2 I,
\]

\[
\dot{\mathbf{C}} = \text{diag}\{0, \mathbf{C}_M\}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{B}_M \end{bmatrix}
\]

with

\[
\mathbf{C}_M = \sqrt{\alpha^2 \mathbf{L} \mathbf{C}},
\]

\[
\mathbf{B}_M = \sqrt{\alpha^2 \mathbf{L} \mathbf{D}}.
\]

**Proof.** Let \( \mathbf{V}(k) = \eta^T(k) \mathbf{P} \mathbf{\eta}(k) \) be the Lyapunov function candidate for the augmented system (6), where \( \mathbf{P} > 0 \).

Calculating the difference of \( \mathbf{V}(k) \) along the system (6) with \( v(k) = 0 \) and taking the mathematical expectation, we have

\[
\begin{align*}
\mathbf{E}\{\Delta \mathbf{V}(k)|x(k); v(k) = 0\} & = \mathbf{E}\{\mathbf{V}(k+1) - \mathbf{V}(k)|x(k); v(k) = 0\} \\
& = \mathbf{E}\{\eta^T(k)A^T \mathbf{P} \eta(k) + \eta^T(k)C^T \mathbf{P} \eta(k) - \eta^T(k)\mathbf{P} \eta(k)\mid x(k); v(k) = 0\} \\
& = \mathbf{E}\{\eta^T(k)(A^T \mathbf{P} \mathbf{A} - \mathbf{P}) \mathbf{\eta}(k) + \eta^T(k)C^T \mathbf{P} \mathbf{C} \mathbf{\eta}(k)\mid x(k); v(k) = 0\}. \\
\end{align*}
\]

From the definition of matrix \( \mathbf{C} \) defined in (6), we can derive

\[
\begin{align*}
\mathbf{E}\{\Delta \mathbf{V}(k)|x(k); v(k) = 0\} & \leq \mathbf{E}\{\eta^T(k)\Lambda_{11} \eta(k)\mid x(k)\} \\
\end{align*}
\]

\[(11)\]

where

\[
\Lambda_{11} \triangleq A^T \mathbf{P} \mathbf{A} - \mathbf{P} + \dot{\mathbf{C}}^T \dot{\mathbf{P}} \mathbf{C}.
\]

In what follows, we are in a position to derive the conditions ensuring the stochastic stability of system (6) with \( v(k) = 0 \). From inequality (8), it can be obtained that \( \Lambda_{11} < 0 \) according to Shur’s complement lemma. It follows from (11) that

\[
\mathbf{E}\{\mathbf{V}(k+1)|x(k)\} - \mathbf{E}\{\mathbf{V}(k)|x(k)\} < 0
\]

which means that the augmented system (6) is stochastically stable.

Now, let us go on the \( H_\infty \) performance analysis. Consider the difference of \( \mathbf{V}(k) \) with nonzero \( v(k) \in l_2[0, +\infty) \).

\[
\begin{align*}
\mathbf{E}\{\Delta \mathbf{V}(k)|x(k)\} & = \mathbf{E}\{\eta^T(k)(A^T \mathbf{P} \mathbf{A} - \mathbf{P}) \mathbf{\eta}(k) \\
& + 2\gamma T(k)A^T \mathbf{P} \mathbf{B}_1 v(k) \\
& + v^T(k)B_1^T \mathbf{P} \mathbf{B}_1 v(k) + v^T(k)B_2^T \mathbf{P} \mathbf{B}_2 v(k) \\
& + \eta^T(k)\mathbf{C}^T \mathbf{P} \mathbf{C} \mathbf{\eta}(k) + 2\gamma T(k)\mathbf{C}^T \mathbf{P} \mathbf{B}_2 v(k)|x(k)\}\}
\end{align*}
\]
Using the same technique of (10), we derive that
\[
\mathbb{E}\{\nu^T(k)B^T\hat{P}B\nu(k)|x(k)\} = \mathbb{E}\{\eta^T(k)C^TP\hat{B}\nu(k)|x(k)\} = \mathbb{E}\{\eta^T(k)\hat{C}^T\hat{P}\nu(k)|x(k)\}.
\]
Let \(\zeta(k) = [\eta^T(k), \nu^T(k)]^T\). Combining (12) and (13), one can easily achieve
\[
\mathbb{E}\{\Delta V(k)|x(k)\} \leq \mathbb{E}\{\chi^T(k)\hat{\Lambda}\zeta(k)|x(k)\}
\]
where
\[
\hat{\Lambda} = \begin{bmatrix} \Lambda_{11} & * \\ * & \Lambda_{22} \end{bmatrix}
\]
with
\[
\Lambda_{22} = B^T\hat{P}B_1 + B^T\hat{P}\hat{B}.
\]
Define an index function:
\[
J = \mathbb{E}\{\Delta V(k)\} + \mathbb{E}\{\|\zeta(k)\|^2\} - \mathbb{E}\{\gamma^2\|v(k)\|^2\}. \tag{15}
\]
We then have
\[
J \leq \mathbb{E}\left\{\sum_{k=0}^{\infty} \eta^T(k)H^T\eta(k) - \gamma^2 v^T(k)v(k) + \chi^T(k)\hat{\Lambda}\chi(k)\right\} = \mathbb{E}\{\sum_{k=0}^{\infty} \chi^T(k)\hat{\Lambda}\chi(k)\} \tag{16}
\]
where \(\hat{\Lambda}\) is defined in (8). If \(\hat{\Lambda} < 0\), then \(J \leq 0\). Under zero initial condition, we then complete the proof by Definition 2.

In the sequel, we will handle the distributed nonfragile \(H_{\infty}\) filtering problem and establish the sufficient condition for the existence of the desired filter. 

**Theorem 3.2** Consider the discrete-time system (1). For a prescribed disturbance attenuation level \(\gamma > 0\), the augmented system (6) satisfies the \(H_{\infty}\) performance constraint (7), if there exist matrices \(P_j > 0, j = 1, 2, \cdots, N+1\), \(X \in \mathbb{W}_{n_x \times n_x}\) satisfying the following linear matrix inequalities (LMIs):
\[
\begin{bmatrix} \Upsilon & \Phi^T \\ * & -P_2 \end{bmatrix} < 0 \tag{17}
\]
where
\[
\Upsilon = \begin{bmatrix} \Upsilon_{11} & 0 \\ 0 & \Upsilon_{22} \end{bmatrix}, \quad \Phi^T = \begin{bmatrix} \sum_{j=1}^{N+1} A^T P_j B & \sum_{j=1}^{N+1} C^T_{M} P_j B_M \\ \sum_{j=1}^{N+1} B^T P_j B + B^T M P_j B_M - \gamma^2 I \end{bmatrix}, \quad P_2 = \text{diag}\{P_2, P_3, \cdots, P_{N+1}\}.
\]
with
\[
\Upsilon_{11} = A^T P_1 A - P_1 + H^T \bar{H}, \quad \Upsilon_{22} = \sum_{j=1}^{N+1} C^T_{M} P_j C_M - P_2 + \bar{H}^T \bar{H}
\]
and
\[
P_2 = \text{diag}\{P_2, P_3, \cdots, P_{N+1}\}.
\]

\(L\) can be determined by
\[
\bar{L} = [c_{ij} L_{ij}]_{N \times N} = P_2^{-1} X. \tag{18}
\]
Thus, we can obtain the filter gains \(L_{ij}(i = 1, 2, \cdots, N, j \in N_+)\) from (5).

**Proof.** Denoting \(P = \text{diag}\{P_1, P_2\}\) and \(P_2 = \text{diag}\{P_2, P_3, \cdots, P_{N+1}\}\), so \(P > 0\) obviously.

By using Schur’s complement lemma, the inequality (8) in Theorem 3 can be converted to
\[
\Psi(\nu) = \begin{bmatrix} \gamma & \Phi^T \\ \Phi & -P_2^{-1} \end{bmatrix} < 0 \tag{19}
\]
where
\[
\Phi^T = \begin{bmatrix} (L_Y \otimes A - L\bar{C} - \bar{\alpha} L_M \bar{C})^T \\ (\bar{B} - L \bar{D} - \bar{\alpha} L_M \bar{D})^T \end{bmatrix}.
\]

Furthermore, pre- and post-multiplying the inequality (19) by \(\text{diag}\{I, P_2\}\) and denoting \(X = P_2 \bar{L}\), we can obtain the inequality (17) in Theorem 3. From Lemma 3, we know \(L = P_2^{-1} X \in \mathbb{W}_n\) due to \(X \in \mathbb{W}_{n_x \times n_x}\). In terms of Theorem 3, the augmented system (6) is stochastically stable with the performance constraint (7). The proof is then completed.

**Remark 3.1** For the augmented system (6), a non-fragile \(H_{\infty}\) filter in the form of (2) is designed in Theorem 3 such that (6) is stochastically stable and the \(H_{\infty}\) performance is satisfied. For the discrete-time stochastic system considered in this paper, there are three main aspects which complicate the design of the \(H_{\infty}\) filters: ROGVs and external disturbances. Different from [26], ROGVs phenomenon is considered when designing filters in this paper. \(\alpha(k)\) is utilized to describe the probabilistic property of such ROGVs.

**4. NUMERICAL EXAMPLE**

This section provides a numerical example to show the effectiveness of the method.

**Example 4.1** Consider a target system (1) with the following parameters:
\[
A = \begin{bmatrix} 0.6 & -0.1 \\ 0.6 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.3 \\ -0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -0.5 \\ -0.4 \\ 0.3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.3 \\ -0.4 \\ 0.2 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.3 \\ -0.4 \\ 0.2 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0.3 \\ -0.4 \\ 0.2 \end{bmatrix}, \quad D_1 = 0.2, \quad D_2 = 0.1, \quad D_3 = 0.2, \quad D_4 = 0.1, \quad H = \begin{bmatrix} -0.4 \\ 0.4 \end{bmatrix}.
\]
The adjacency matrix is
\[
C = \begin{bmatrix}
1 & 0.9 & 0.7 & 0 \\
0 & 1 & 0 & 0.8 \\
0.8 & 0 & 1 & 0.9 \\
0.7 & 0 & 0 & 1
\end{bmatrix}.
\]

Assume
\[
M_{11} = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, \quad M_{12} = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \quad M_{13} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},
\]
\[
M_{22} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \quad M_{24} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \quad M_{31} = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},
\]
\[
M_{33} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \quad M_{34} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad M_{41} = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix},
\]
\[
M_{44} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.
\]

Assuming the stochastic variable \( \alpha(k) \) obeys Bernoulli distribution and \( \bar{\alpha} = 0.80 \). Then the minimum disturbance attenuation level is \( \gamma_{\text{min}} = 0.3661 \).

Under the same conditions, we set the guaranteed \( H_{\infty} \) norm \( \gamma = 1.4 \). And based on Theorem 3, we can find the feasible solutions
\[
L_{11} = \begin{bmatrix} -1.1618 \\ -1.0814 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} -0.2723 \\ -0.3648 \end{bmatrix},
\]
\[
L_{13} = \begin{bmatrix} 0.2301 \\ -0.4623 \end{bmatrix}, \quad L_{22} = \begin{bmatrix} -0.8348 \\ -0.6847 \end{bmatrix},
\]
\[
L_{24} = \begin{bmatrix} -0.2120 \\ -0.1962 \end{bmatrix}, \quad L_{31} = \begin{bmatrix} -0.0629 \\ -0.3295 \end{bmatrix},
\]
\[
L_{33} = \begin{bmatrix} -1.5035 \\ -1.5477 \end{bmatrix}, \quad L_{34} = \begin{bmatrix} -0.0726 \\ -0.3428 \end{bmatrix},
\]
\[
L_{41} = \begin{bmatrix} -0.1211 \\ -0.2443 \end{bmatrix}, \quad L_{44} = \begin{bmatrix} -1.3725 \\ -1.1729 \end{bmatrix}.
\]

Given the initial values of the state and its estimate as \( x(0) = [2.4, -2.5]^T \) and \( \hat{x}(0) = [2.0, -2.9]^T \), the response curves of \( x(k) \) and their estimates \( \hat{x}(k)(i = 1; 2; 3; 4) \) are shown in Fig. 1 and Fig. 2. And, the estimation errors dynamics are shown in Fig. 3 which can clearly reflect the desired convergence of the estimation errors under the designed distributed filters.

**5. CONCLUSIONS**

This study has focused on the problem of the distributed non-fragile \( H_{\infty} \) filtering of discrete-time systems with randomly occurring gain variations. A stochastic variable \( \alpha(k) \) has been utilized to model the probabilistic property of the randomly occurring gain variations. By utilizing the Lyapunov function method, a sufficient condition which can deal with the stochastic stability for a class of discrete-time systems with randomly occurring gain variations has been proposed. Moreover, a distributed non-fragile \( H_{\infty} \) filter has been designed for discrete-time systems over sensor networks. Finally, an illustrative numerical example has been given to verify the usefulness of this method.

**REFERENCES**


