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Cooperative Control of CAVs Using Control Barrier Function at Signal-free Intersection Considering Real Environment

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Abstract: This study addresses the cooperative control of CAVs(Connected and Automated Vehicles) at Signal-free Intersection. The purpose is to reduce fuel consumption by crossing intersection without stopping as much as possible while ensuring safety. For this problem, We consider a two-stage control structure.The first stage is to calculate the merging time of each CAV by solving mixed integer linear programming (MILP). In the second stage, each CAV solves the optimal control problem and determines the input in time for the merging time. In this study, We propose an optimal control method using control barrier function considering more realistic scenario, and a speed regulation zone to cross the intersection at a pre-determined terminal speed. By using input corrections and trajectory tracking of speed and position, simulation results show that fuel consumption can be reduced while ensuring safety. Finally, compared to conventional research, the system achieves low fuel consumption, and demonstrating its superiority.

Keywords: Control Barrier Function, Optimal Control, Signal-free Intersection, CAV

1. INTRODUCTION

Traffic management at intersections is important in terms of accidents and congestion on public roads. 2,583 traffic fatalities occurred in Japan in 2021, 46.6 % of which were intersection-related [1]. The main cause of these fatalities is human error. Currently, traffic signals are the main cause of accidents and congestion at intersections, and there are many problems to be solved.

CAVs (Connected and Automated Vehicles) is expected to be put into practical use. CAVs can communicate with each other and with the infrastructure to adjust the timing of crossing at intersections, and are expected to make intersections unsignalized. Signal-free Intersection is expected to reduce traffic congestion and improve throughput by reducing unnecessary stops, and reduce traffic accidents by reducing human error through the use of automated CAVs.

In general, control at Signal-free Intersection is a two-stage problem, with the first stage determining the order and time to cross the intersection, and the second stage determining the control inputs for the vehicle to meet that order and time. For the first stage, the first-in-first-out (FIFO) method is often used to determine the crossing order from a fairness perspective [2] , while the mixed integer linear programming (MILP) method is used to determine the crossing order and time from an efficiency perspective [3, 4]. Other methods include game theory [5] . For the second step, optimal control [2] and model predictive control [6, 7] are used to determine the control inputs. For optimal control, using control barrier functions has been active in recent years [8, 9]. The advantage of control barrier functions is that they can determine control inputs without increasing the computational load, even for complex dynamics.

Based on the above studies, this study proposes a more realistic cooperative control method for CAVs at Signal-free Intersections that can consider air resistance and road surface conditions by using control barrier functions and disturbance observers. In addition, a speed regulation zone is established to allow the vehicle to cross the intersection at a defined terminal speed. The proposed method also allows vehicles to safely cross the intersection by avoiding crossing at unrealistic speeds due to the emphasis on efficiency.

† Yuta Nakai is the presenter of this paper.

2. PROBLEM FORMULATION

2.1. Intersection Model

Consider the single-lane intersection model shown in Fig. 1. The Control Zone (CZ) is defined as the area

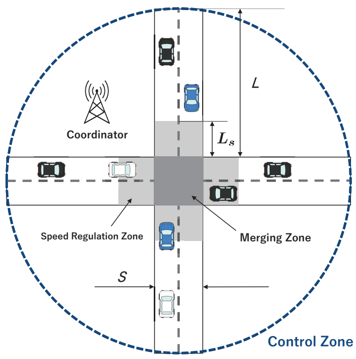

Fig. 1 Intersection Model

within which the CAVs can communicate with the Coordinator. The area where CAVs merge is defined as the Merging Zone (MZ), and the Speed Regulation Zone (SRZ) is defined as the area where speed is regulated to enter the MZ at a terminal speed.

In the MZ, Conflict Point (CP) is set up as shown in the following Fig. 2. CP refers to the point at which CAV_i is likely to conflict with CAV_j entering the MZ from another lane when crossing the MZ. Also, let the lanes be denoted by lane1∼4, respectively.

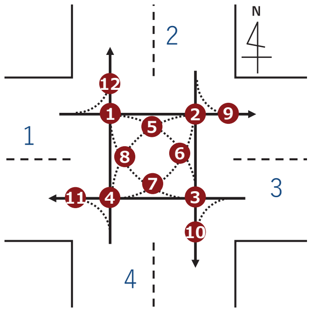

Fig. 2 Conflict Point

2.2. Vehicle Model

The vehicle Model used in this study is considered to be a model with a torque τ as shown in Fig. 3.

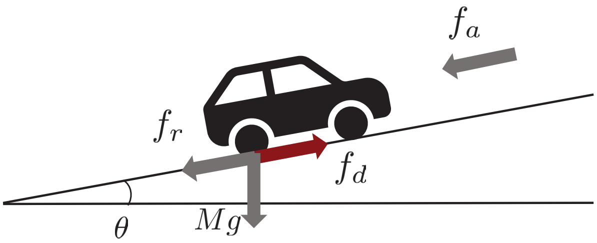

Fig. 3 Vehicle Model

The equation of motion is

$$M\dot{v}(t) = f_d(t) - f_a(t) - f_r(t) - f_g(t) \tag{1}$$

where

$$\begin{aligned} f_d(t) &= \frac{\tau(t)}{R_w}, \quad & f_a(t) &= \frac{1}{2}\rho A_f C_d v^2(t) \\ f_r(t) &= \mu M g \cos\theta, \quad & f_g(t) &= M g \sin\theta \end{aligned} \tag{2}$$

where f_d is the driving force of the vehicle, f_a is air resistance , f_r is rolling resistance, and f_g is slope resistance. M is the vehicle weight and R_w is the tire radius. Explanations for each letter are given below. The vehicle is equipped with automatic driving and communication functions, and is configured as a CAV to perform cooperative control using multiple vehicles.

Table 1 Symbol

Symbols	Definition
M	Weight of vehicle [kg]
R_w	Tire radius [m]
ρ	Air density [kg/m^3]
C_d	Air resistance coefficient
A_f	Vehicle frontal area [m^2]
μ	Rolling resistance coefficient
θ	Road tilt angle [rad]

From the equations of motion, we have $\frac{1}{2}\rho A_f C_d v_i^2(t) + \mu M g \cos\theta + M g \sin\theta = F(v_i(t))$ to represent the state space model of some CAV$_i$ as

$$\begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t) \\ -\frac{F(v_i(t))}{M} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{MR_w} \end{bmatrix} \tau_i(t) \tag{3}$$

Where p_i is the position of CAV, v_i is the speed of CAV. In addition, from the viewpoint of vehicle performance and passenger comfort, the following input and speed constraints are set as shown in (4) below.

$$\begin{aligned} \tau_{min} &\leq \tau_i(t) \leq \tau_{max} \\ v_{min} &\leq v_i(t) \leq v_{max} \end{aligned} \tag{4}$$

Let us assume that the following assumptions hold for CAV.

Assumption 1: At Signal-free Intersection, only CAVs of at least Level 4 shall be present, with no communication problems such as errors or delays, pedestrians, and all CAVs shall have the same width and length, mass, and equipment.

2.3. Control Structure

In this study, Cooperative Control is divided into two stages: one in which the Coordinator determines the optimal merging time to enter the SRZ based on the information from each CAV, and the other in which the CAV determines the control inputs so that it can cross the MZ at a predetermined speed and arrive at the SRZ at the merging time. In this case, we consider the control structure shown in the following Fig. 4.

where $Y_i^s \triangleq \{\eta_i, p_i, v_i, t_{i,prev}^m\}$ is the dataset sent from CAV$_i$ to Coordinator, $Y_i^r \triangleq \{t^{m*}_i, p_i(t^{m*}_i), v_i(t_i^{m*})\}$ is the dataset sent from Coordinator to CAV$_i$. Also, η_i is the travel lane t_i^{m*} is the optimal merging time calculated by the Coordinator and $t_{i,prev}^m$ is the merging time calculated by the Coordinator at the previous sampling time.

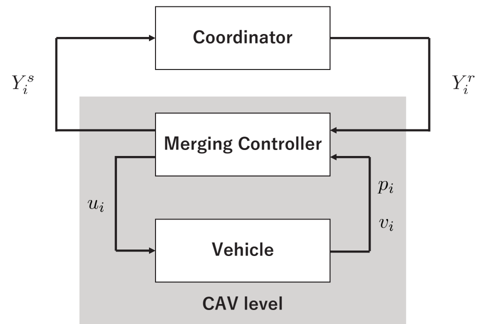

Fig. 4 Control Structure

2.4. Speed Regulation Zone

SRZ performs deceleration using a predetermined speed regulation time t_i^{sr} as shown in Fig. 5.

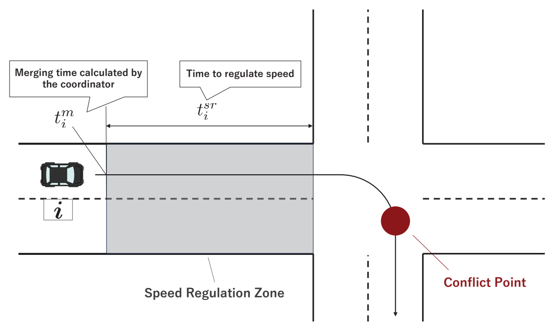

Fig. 5 Speed Regulation Zone

In doing so, the Coordinator uses the right/left turn information from the CAV to regulate the speed to a predetermined terminal speed. Fixing the end speed prevents the vehicle from entering the MZ at an unrealistic speed.

3. COORDINATOR-LEVEL OPTIMIZATION

This chapter shows how Coordinator determines the optimal merging time for each CAV, which is the time at which each CAV crosses the intersection as soon as possible to avoid conflicts with other CAVs.

3.1. Constraints

First, consider the minimum merging time calculated by Coordinator to avoid missing the merging time due to CAV performance. Calculate the time it takes for the CAV to accelerate with the maximum torque and decelerate with the minimum torque to reach the terminal speed v_m. Using the distance $D = L - L_s - p_i(t)$ from the current position $p_i(t)$ to the SRZ

$$
\begin{aligned}
t^m_{i,min} =& \frac{MR_w(v_{max} - v_i)}{\tau_{max}} + \frac{MR_w(v_m - v_{max})}{\tau_{min}} \\
&+ \frac{D - \frac{MR_w(v_{max}^2 - v_i^2)}{2\tau_{max}} - \frac{MR_w(v_m^2 - v_{max}^2)}{2\tau_{min}}}{v_{max}} \dots (a) \\
t^m_{i,min} =& \frac{MR_w(v' - v_i)}{\tau_{max}} + \frac{MR_w(v_m - v')}{\tau_{min}} \\
&\text{where} \quad v' = \sqrt{\frac{\frac{2D}{MR_w} + \frac{{v_i}^2}{\tau_{max}} - \frac{{v_m}^2}{\tau_{min}}}{\frac{1}{\tau_{max}} - \frac{1}{\tau_{min}}}} \dots \quad (b)
\end{aligned}
\tag{5}
$$

(a) is the case where the maximum speed in the equation (4) is reached between the current position and the entrance to the SRZ. When the maximum speed is reached, the vehicle cannot accelerate anymore, so it runs at a constant speed and then decelerates so that it can eventually enter the SRZ at the terminal speed. (b) is the case where the maximum speed is not reached. In this case, the vehicle does not travel at a constant speed, but accelerates at maximum torque and decelerates at minimum torque to enter the SRZ at the terminal speed.

Next, constraints are set to avoid a rear-end headway with a preceding vehicle (CAV$_k$) in the same lane. rear-end headway constraint as

$$
t^m_i - t^m_k \geq \frac{\alpha_i(t)}{v_{m,k}} = h_R, \quad \eta_k = \eta_i \tag{6}
$$

where $\alpha_i(t) = \delta + l_k + \varphi v_i(t)$. δ is the safety distance between the CAV$_k$ rear bumper and the CAV$_i$ front bumper, φ is the reaction time of CAV$_i$, and l_k is the length of the CAV$_k$. η is a lane of CAVs. With this constraint, rear-end headway can be avoided in the MZ. In order to avoid rear-end headway in the SRZ, a speed regulation time t^{sr}_i is added to the equation (6). The equation for the rear-end headway constraint is

$$
(t^m_i + t^{sr}_i) - (t^m_k + t^{sr}_k) \geq h_R \tag{7}
$$

The rear-end headway constraint here is to prevent rear-end headway from reaching the SRZ until exiting the MZ. In other words, rear-end headway on the road are not considered. Therefore, it is necessary for CAVs to communicate with each other to avoid rear-end headway along the way, independently of the Coordinator.

Next, consider a constraint to avoid a lateral headway with a CAV running in a different lane. The likelihood of a headway is determined from Fig. 2. As shown in Fig. 6, the Conflict Zone is a circle of radius r centered at the CP. The time taken by the CAV$_i$ to enter the Conflict Zone after entering the MZ is t^e_i, the time taken by the CAV$_i$ to exit the Conflict Zone after entering the MZ is Let t^l_i be the time from entering the MZ to exiting the Conflict Zone.

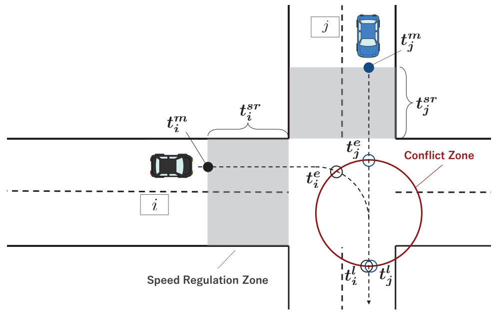

Fig. 6 Lateral Headway Time

Using these to consider the lateral headway constraints, we consider the speed regulation time t^{sr}_i as well as the rear-end headway constraints, and we obtain

$$
\begin{aligned}
&(t^m_i + t^e_i + t^{sr}_i) + M_{big} b_{ij} \geq (t^m_j + t^l_j + t^{sr}_j) + h_L \\
&\text{AND} \\
&(t^m_j + t^e_j + t^{sr}_j) + M_{big}(1 - b_{ij}) \geq (t^m_i + t^l_i + t^{sr}_i) + h_L
\end{aligned}
\tag{8}
$$

where h_L is the time required for the CAV to completely exit the Conflict Zone. Since the position of the CAV is assumed to be that of the front bumper in this case, the time until the rear bumper of the CAV completely exits the Conflict Zone is assumed to be the time.

In addition, if the equation is expressed with OR, it becomes discontinuous, so it is expressed with an AND equation using the Big-M method. where b is a binary variable.

In summary, the optimal merging times calculated by Coordinator can be obtained by solving

$$
\begin{aligned}
\min_{T,B,T^{abs}} \quad & w \sum_{i=1}^{N(t)} t^m_i + (1 - w) \sum_{i=1}^{N(t)} t^{abs}_i \\
\text{subject to:} \quad & t^m_i \geq t^m_{i,min} \\
& (t^m_i + t^{sr}_i) - (t^m_k + t^{sr}_k) \geq h_R \quad \eta_i = \eta_k \\
& (t^m_i + t^e_i + t^{sr}_i) + M_{big} b_{ij} \\
& \qquad \geq (t^m_j + t^l_j + t^{sr}_j) + h_L \\
& (t^m_j + t^e_j + t^{sr}_j) + M_{big}(1 - b_{ij}) \\
& \qquad \geq (t^m_i + t^l_i + t^{sr}_i) + h_L \\
& t^{abs}_i \geq (t^m_i - t^m_{i,prev}) \\
& t^{abs}_i \geq -(t^m_i - t^m_{i,prev})
\end{aligned}
\tag{9}
$$

where t^{abs}_i is the difference from previous merging time, and the purpose of this term is to avoid abrupt changes in the merging time. And T is a set of t^m_i, T^{abs}_i is a set of t^{abs}_i. $N(t)$ is the total number of CAVs in the CZ. Also, B is a set of binary variables b_{ij} to determine the order in which CAV$_i$ and CAV$_j$ intersect.

4. CAV-LEVEL OPTIMIZATION

In this section, We formulate the problem of determining the control input by the CAV by solving the optimal control problem based on the optimal merging time calculated by Coordinator. In addition to speed and input constraints, a cooperative control framework is presented to allow the CAV to cross an intersection without conflict with other CAVs .

4.1. Formulation of the optimal control problem

The optimal control problem solved by each CAV is as follows .

$$\begin{aligned} &\min_{\tau_i(t)} && \int_{t_i^0}^{t_i^m} \frac{1}{2}\tau_i^2(t)dt \\ &\text{subject to:} && \text{CAV model(3)} \\ & && \text{Input and Speed Constraint(4)} \\ & && p_k(t) - p_i(t) \geq \varphi v_i(t) + \delta \\ & && \text{given } \; p_i(t_i^0)\,,\; v_i(t_i^0)\,,\; v_i(t_i^m)\,,\; p_i(t_i^m) \end{aligned} \tag{10}$$

The constraints are an input constraint, a speed constraint, and a rear-end headway constraint to prevent conflict on the road. The t_i^0 is the current time.

We want to solve the optimal control problem and obtain an analytical solution, but it is difficult to obtain an analytical solution because v_i is included in the resistance term such as air resistance in the (10). Therefore, the analytical solution in the ideal state with no resistance is obtained, and the obtained analytical solution is followed.

$$\tau_i^* = C_1 t + C_2 \tag{11}$$

$$v_i^* = \frac{1}{MR_w}\left(\frac{1}{2}C_1 t^2 + C_2 t + C_3\right) \tag{12}$$

$$p_i^* = \frac{1}{MR_w}\left(\frac{1}{6}C_1 t^3 + \frac{1}{2}C_2 t^2 + C_3 t + C_4\right) \tag{13}$$

where C_1, C_2, C_3, C_4 are integral constants and the boundary conditions $p_i(t_i^0)$, $v_i(t_i^0)$, $v_i(t_i^m)$, $p_i(t_i^m)$ can be obtained using the following equation.

4.2. Input Correction

In the real environment, even if the analytical solution is used, the speed and position will deviate from the trajectory when there is a resistance such as (2). Therefore, in order to eliminate modeling errors, we consider a method to correct the input using a disturbance observer such as the following .

$$\tau_i^{ref}(t) = \tau_i^*(t) + \hat{d}_i \cdot R_w \tag{14}$$

where $\hat{d}_i$ is the estimated resistance $F(v_i(t))$. The purpose of this disturbance observer is to estimate the resistance $F(v(t))$, and add it to the input to cancel resistance out and track the trajectory.

4.3. Formulation with Control Barrier Function

For these constraints in (10), the input τ_i is not included in the constraint equation. It is difficult to determine the inputs such that the constraints are not violated if they become active. Therefore, we consider a formulation with a control barrier function. For simplicity, the vehicle model (3) is

$$\dot{x} = f(x) + g(x)u \tag{15}$$

Then define the control barrier function. First, we define the safety set.

Definition 1: safety set of CAV$_i$

Consider a C^1 class function $h : X \to \mathbb{R}$ and a set $\mathcal{C} \subset X$ expressed as follows.

$$\mathcal{C} = \{x_i \in X \mid h(x_i) \geq 0\} \tag{16}$$

If the state x_i of CAV$_i$ satisfies $x \in \mathcal{C}$, then $\mathcal{C}$ is a safe set.

This safety set is only for CAV$_i$. For the rear-end headway constraint, information on the preceding vehicle, CAV$_k$, is also required, so a 2 CAVs safety set is also needed to be defined.

Definition 2: safety set of CAV$_i$, CAV$_k$

Consider a C^1 class function $H : X_i \times X_k \to \mathbb{R}$ and a safe set $\mathcal{C}' \subset X_i \times X_k$ expressed as follows.

$$\mathcal{C}' = \{(x_i, x_k) \in X_i \times X_k \mid H(x_i, x_k) \geq 0\} \tag{17}$$

Next, we define a forward invariant set obtained from a safe set.

Definition 3: forward invariant set

A set $\mathcal{C}$ is said to be a forward invariant set if the initial state $x(0)$ of the system (3) is contained in the set $\mathcal{C}$ and $x \in \mathcal{C}$ at any time $t \in [0, \infty)$.

This forward invariant set can likewise be defined for the safe set of 2 CAVs. Next, we define the extended class $\mathcal{K}$ function.

Definition 4: extended class $\mathcal{K}$ function

Consider a continuous function $\alpha : \mathbb{R} \to \mathbb{R}$. When this function is strictly monotonically increasing and satisfies $\alpha(0) = 0$, it is called an extended class $\mathcal{K}$ function.

Finally, we define the control barrier function.

Definition 5: control barrier function

Let h be a C^1 function and let $\mathcal{C} \subset X$ be the safety set of CAV$_i$ for the vehicle model (3). If there exists an extended class $\mathcal{K}$ function α that satisfies the following inequality (18), then the function h is called a control barrier function defined on $\mathcal{C}$.

$$\sup_{u_i \in U} \dot{h}(x_i) + \alpha(h(x_i)) \geq 0 \tag{18}$$

Here, $\dot{h}(x_i)$ can be expressed as $\dot{h}(x_i) = L_f h(x_i) + L_g h(x_i) u_i$ by the Lie derivative We can define the following set $K_i(x_i)$ from this control barrier function h.

Definition 6: $K_i(x_i)$

$$K_i(x_i) = \{u_i \in U : \dot{h}(x_i) + \alpha(h(x_i)) \geq 0\} \tag{19}$$

Definition 7: control barrier function

Let $\mathcal{C}'$ be the safety set of CAV$_i$ and CAV$_k$, and let $H : X_i \times X_k \to \mathbb{R}$ be a C^1 function. If there exists an extended class $\mathcal{K}$ function that satisfies the following inequality for the safety set $\mathcal{C}'$, H is called a control barrier function.

$$\sup_{u_i \in U} \dot{H}(x_i, x_k) \geq -\alpha(H(x_i, x_k)) \tag{20}$$

We can define the following set $K_H(x)$ from this control barrier function H

Definition 8: $K_H(x)$

$$K_H(x) = \{u \in U : \dot{H}(x_i, x_k) \geq -\alpha(H(x_i, x_k))\} \tag{21}$$

A control barrier function is then defined. Using the input obtained from the control barrier function, it can be said that the input stays within the safe set, i.e., forward invariance is satisfied.

Theorem 1: Let $\mathcal{C}$ be a safe set of CAV$_i$. When $h(x_i)$ is a control barrier function, the Lipschitz continuous input $u \in K_i(x_i)$ satisfies forward invariance on the safe set.

Proof: Since $\dot{h}(x_i)$ is always greater than or equal to 0 at the boundary of the safe set, it satisfies forward invariance by Nagumo's theorem [10]. ■

Theorem 2: Let $\mathcal{C}'$ be the safe set of CAV$_i$ and CAV$_k$, and $H : X_i \times X_k \to \mathbb{R}$ be a C^1 class function. When H is a control barrier function, the Lipschitz continuous $u_i \in K_H(x_i, x_k)$ satisfies forward invariance on the safe set.

Proof: We consider $z = \begin{bmatrix} x_i \\ x_k \end{bmatrix}, u' = \begin{bmatrix} u_i \\ u_k \end{bmatrix}$,

$F(z) = \begin{bmatrix} f(x_i) \\ f(x_k) \end{bmatrix}, G(z) = \begin{bmatrix} g(x_i) \\ g(x_k) \end{bmatrix}$

we can set $H(x_i, x_k) = H(z)$, which satisfies forward invariance by Nagumo's theorem as well as Theorem 1. [9, 11]. ■

Therefore, it is confirmed that forward invariance is satisfied. Using this control barrier function, the optimal control problem (10) can be rewritten

$$
\begin{aligned}
\min_{\tau_i(t)} \quad & \int_{t_i^0}^{t_i^m} \frac{1}{2}\left(\tau_i^{ref}(t) - \tau_i(t)\right)^2 dt \\
\text{subject to:} \quad & \text{CAV model(3)} \\
& \tau_i - \tau_{min} \geq 0 \\
& \tau_{max} - \tau_i \geq 0 \\
& -\frac{F(v_i(t))}{M} + \frac{1}{MR_w}\tau_i(t) + v_i(t) - v_{min} \geq 0 \\
& \frac{F(v_i(t))}{M} - \frac{1}{MR_w}\tau_i(t) + v_{max} - v_i(t) \geq 0 \\
& v_k(t) - v_i(t) + \frac{F(v_i(t))}{M} - \frac{\varphi}{MR_w}u_i(t) \\
& \quad + x_k(t) - x_i(t) - \varphi v_i(t) - \delta \geq 0 \\
& \text{given } \; p_i(t_i^0)\,,\; v_i(t_i^0)\,,\; v_i(t_i^m)\,,\; p_i(t_i^m)
\end{aligned}
\tag{22}
$$

The objective function $\left(\tau_i^{ref}(t) - \tau_i(t)\right)^2$ is used to track the torque τ_i^{ref} after input correction.

4.4. Optimal control problem in Speed Regulation Zone

Finally, there is the optimal control problem in SRZ. speed regulation time t^{sr} is determined in advance, and rear-end headway in SRZ are taken into account by Coordinator. Therefore, the optimal control problem solved by CAVi is the expression that eliminates the rear-end headway constraint from (22).

$$
\begin{aligned}
\min_{\tau_i(t)} \quad & \int_{t_i^m}^{t_i^m + t_i^{sr}} \frac{1}{2}\left(\tau_i^{ref}(t) - \tau_i(t)\right)^2 dt \\
\text{subject to:} \quad & \text{CAV model(3)} \\
& \text{input constraints(22)} \\
& \text{speed constraints (22)} \\
& \text{given } \; p_i(t_i^m)\,,\; v_i(t_i^m)\; v_i(t_i^m + t_i^{sr})\,,\; p_i(t_i^m + t_i^{sr})
\end{aligned}
\tag{23}
$$

5. SIMULATION VERIFICATION

5.1. Simulation Conditions

In order to verify the simulation, we set the parameters as in Table 2.

Table 2 Simulation parameter

Parameter	Symbol	Value
Number of vehicles	N_{max}	50
Vehicle arrival rate(1/h)	f^a	720
Control Zone(m)	L	300
Speed Regulation Zone(m)	L_s	30
Size of Merging Zone(m)	S	10
Radius of conflict point zone(m)	r	4
Weight of MILP	w	0.3
Length of CAV(m)	l_i	4
Standstill safe distance(m)	δ	8
CAV reaction time(s)	φ	0.8
Speed Regulation time(straight,right,left)(s)	$t_S^{sr}, t_R^{sr}, t_L^{sr}$	3.0,3.6,4.5
Terminal Speed (straight,right,left)(m/s)	v_S, v_R, v_L	10,7,3
Speed Constrains(m/s)	v_{min}, v_{max}	0,16
Input Constraints (kgm^2/s^2)	u_{min}, u_{max}	-1200,1200

The slope of lane1 (lane from the west) and lane2 (lane from the north) is set to $1°$, and the slope of lane3 (lane from the east) and lane4 (lane from the south) is set to a downhill slope of $-1°$. The poles of the disturbance observer are set to [-84,-85,-86].

5.2. Simulation Results

These graphs of the simulation results are shown below: Fig. 7 shows the input of each CAV, and Fig. 8 shows the speed of each CAV. Fig. 9 shows the position of each CAV. The center of the position in Fig. 9 is the MZ, and the top or bottom is the entrance to the CZ. Fig. 9 shows the trajectory of each CAV from its entry into the CZ to its arrival at the center MZ. In addition, the trajectory is drawn from the top when coming from the north and east, and from the bottom when coming from the south and west.

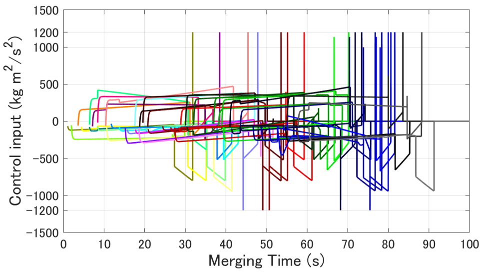

Fig. 7 Input for each CAV

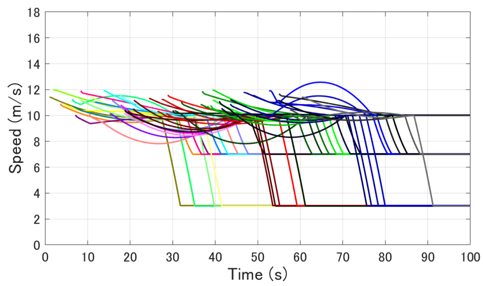

Fig. 8 Speed for each CAV

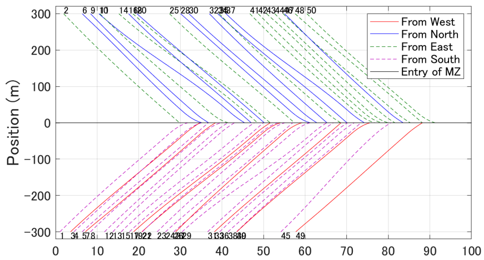

Fig. 9 Position for each CAV

Fig. 7 shows that the input constraints are obeyed. Fig. 8 shows that the speed constraint is satisfied. The terminal speed shows that there are three speeds: go straight, turn left, and turn right, respectively. Fig. 9 shows that there is no conflict before passing through the MZ.

The formulation of optimal control using the control barrier function allows for cooperative control without violating arbitrary constraints. In addition, the SRZs allowed the vehicles to pass through the intersection at a defined terminal speed.

Finally, we compare the fuel consumption. For fuel consumption, we use the polynomial model proposed in [8] and [9] are used for comparison. Both of them use optimal control with control barrier functions as a method. The method used in [8] is to correct input by expressing the position of the optimal solution and the current position in terms of fractions. The method is expressed as

$$\tau_i^{ref}(t) = \frac{p_i^*(t)}{p_i(t)} \tau_i^*(t) \tag{24}$$

Next, the method used in [9] uses PI control from the speed to correct the input. The difference between the speed and position of the optimal solution and the current speed and position is used. The method is expressed as

$$\tau_i^{ref}(t) = \tau_i^*(t) + K_p(p_i^*(t) - p_i(t)) + K_v(v_i^*(t) - v_i(t)) \tag{25}$$

The proposed method compares these methods with a method without input correction (None). The proposed method uses a disturbance observer to correct input, as shown in (14). The fuel consumption values are a summary of the ones of 50 CAVs. And the other parameters are the same as in Table 2. In this study, we compare these input correction methods. Since these literatures are based on double lines and direct comparison is not possible, only the input correction methods are compared. Also, the gains K_p, K_v of Pi control are 35000 and 30000, respectively.

The results are shown in the following table.

Table 3 Fuel Consumption

Method	Fuel Consuption(L)
Proposed	0.9098
Fractional type [8]	0.9758
PI control [9]	0.9332
None	0.9758

It can be seen that the proposed method has the lowest fuel consumption. The results of Document A are not significantly different from those of the case without input correction. In the case of the B, it can be seen that the fuel consumption is reduced by the input correction.

6. CONCLUSION

In this study, we proposed an algorithm for cooperative control of CAVs at Signal-free Intersection that takes into account more realistic conditions such as air resistance and road surface conditions. Specifically, we proposed a safe optimal control method that does not violate constraints using a control barrier function, an input correction method using a disturbance observer that can cope with modeling errors, and a method that can cross an intersection without conflicts while maintaining the terminal speed by setting a deceleration zone at a predetermined speed.

Future challenges include making the system operable in mixed environments with human-driven vehicles and reducing the computational load, and decentralizing the system.

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